DOI:10.22600/1518-8795.ienci2021v26n3p82

ienci Investigações em Ensino de Ciências
V26 (3) – Dez. 2021
pp. 82 - 101

CONCEPTUAL FIELDS AND MAGNETIC FIELD: A THEORETICAL MODEL FOR EPISTEMOLOGICAL CLASSIFICATION OF TASKS IN MAGNETOSTATICS

Campos conceituais e campo magnético: um modelo teórico para classificação epistemológica de tarefas em magnetostática

Glauco Cohen Ferreira Pantoja [glauco.pantoja@ufopa.edu.br]
Instituto de Ciências da Educação
Universidade Federal do Oeste do Pará
Avenida Marechal Rondon S/N, Santarém, Pará, Brasil

Abstract

This work presents a theoretical model for epistemological classification of tasks in magnetostatics aimed at High School and Higher Education. The approach is based on the theory of conceptual fields and includes classification in terms of thought operations necessary to solve the tasks and in these situations' parameters. Four primary classes of situations are proposed, namely, description of magnetic interactions, analogic symbolization of magnetic fields, non-analogic symbolization of magnetic fields and calculation of magnetic fields. These classes cannot be reduced one to another, however they can occur simultaneously in the same task. Each one was subdivided in secondary classes of situations based on parameters they can assume and ordered by epistemological complexity. As contributions for physics teaching research this work offers a theoretical-methodological model for analyzing students' progression in the conceptual field of magnetostatics, a conceptual structure for building situations based on predicative and operational competences for understanding the concept of magnetic field, and a practical example of epistemological classification of situations that can be adapted for other areas of Science like Quantum Mechanics, for example.

Keywords: Conceptual Fields; Magnetic Field; Classification of Tasks.

Resumo

Esse trabalho apresenta um modelo teórico para classificação de tarefas em magnetostáticas que foca no Ensino Médio e no Ensino Superior. A abordagem é baseada na teoria dos campos conceituais e inclui a classificação em termos de operações de pensamento necessárias à resolução das tarefas e aos parâmetros dessas situações. Quatro classes primárias de situações são propostas, a saber, descrição de interações magnéticas, simbolização analógica de campos magnéticos, simbolização não analógica de campos magnéticos e cálculo de campos magnéticos. Essas classes não podem ser reduzidas umas às outras, embora elas possam ocorrer simultaneamente na mesma tarefa. Cada classe primária foi dividida em classes secundárias de situações baseadas nos parâmetros que elas podem assumir e ordenadas por complexidade epistemológica. Como contribuições para a pesquisa em ensino de Física, esse trabalho oferece um modelo teórico-metodológico para análise da progressão dos estudantes no campo conceitual da magnetostática, uma estrutura conceitual para construir situações com base em competências predicativas e operatórias para entender o conceito de campo magnético, e um exemplo práticos de classificação epistemológica de situações que pode ser adaptado para outras áreas da Ciência como a Mecânica Quântica, por exemplo.

Palavras-chave: Campos Conceituais; Campo Magnético; Classificação de tarefas.

INTRODUCTION

Processes of teaching and learning the concept of magnetic field have been objects of research throughout time and results point to huge difficulties due to teachers in teaching this content and students in learning it (Zuza, Van Kampen, de Cock, Kelly, & Guisasola, 2018; Mbonyiryivuze, Yadav & Amadalo, 2020; Zuza, Almudí, & Guisasola, 2012; Pantoja & Moreira, 2019b). The background does not seem to change from High School to Higher Education, including courses of teacher training. This may indicate conceptual learning is perhaps better understood in a developmental approach (Vergnaud, 2013).

A considerable quantity of comprehension difficulties faced by students on electromagnetism seem be related with a methodological change from mechanics to electromagnetic theory (Galili, 1995; Guisasola, Almudí, & Zubimendi, 2004; Zuza et al, 2018), because the latter considers the electromagnetic field as a mediator of energy and momentum exchanges between electrically charged objects. The epistemological role of the field in electromagnetic theory is central for its comprehension, once it puts this paradigm in a coherent whole. In other words, the problem seems to be also epistemological in its nature.

Nousianen and Koponen (2017) state it is possible to comprehend electromagnetism epistemologically by means of three facets, the ones of force, energy and source. The force and energy facets aim to describe electromagnetic interactions as momentum and energy exchanges mediated by the electromagnetic field, whilst the third is oriented to indicate the generation of electromagnetic fields by electrically charged objects. In terms of conceptual learning, beginners usually seem to assimilate better the force facet than the other two (Nousianen & Koponen, 2017).

Although the force facet seems to be the easiest one to be learned, this does not mean students do not have any difficulty at all in doing it. There are also other kinds of confusion that university students establish between electric and magnetic forces, and these may be associated to proactive and retroactive interference (Scaife & Heckler, 2011). Students usually attribute the same direction of magnetic force and magnetic field when magnetostatics is taught after electrostatics (proactive interference) or students indicate perpendicularity to directions of electric field and electric force when the magnetosticatic is taught before electrostatics (retroactive interference). Results expressing misunderstanding between electricity and magnetism are found in other works and can be generalized for diverse European and African contexts (Zuza *et al.*, 2018; Nousianen & Koponen, 2017).

Specific difficulties in comprehending the sources of the electromagnetic field can also be found in literature, for example, they are commonly related to the application of the concepts of divergent and rotational to study electromagnetic (Bollen, Van Kampen, Baily & de Cock, 2016; Guisasola, Almudí, Salinas, Zuza & Ceberio, 2008). Moreover, a plenty of students are susceptible to experience difficulties in interpreting graphic representations related to vector fields and in identifying sources of magnetic fields (Guisasola, Almudí, & Zubimendi, 2004; Brandamante & Viennot, 2007; Campos, Zavala, Zuza, & Guisasola, 2020). Thus, there are myriads of misconceptions held by students in the three cited domains.

Recently, physics teaching research have brought didactical strategies deemed positive for addressing students' hindrances (Pantoja, 2015), although none of them has explicitly considered students' operatory knowledge, that one used when actions are carried out (Vergnaud, 2013). The results of these studies often emphasize explicit and symbolic forms of knowledge built by students, but conceptual learning depends on both predicative (linguistic) and operational (action) forms of knowledge for happening (Vergnaud, 2012). Then, planning teaching strategies must focus on these two instances at the same time.

We take the context of magnetostatics, for instance, to clarify the importance of operational knowledge, which is used and developed in situations (Vergnaud, 2009), for learning. For example, hindrances in comprehension of the direction of the magnetic force exerted by a uniform magnetic field on a moving electric charge were commonly detected by prior research (Scaife & Heckler, 2011; Guisasola, Almudí & Zubimendi, 2004). These obstacles seem to become more intense as long as instruction is implemented, because after some time, the fact of turning explicit a source of magnetic field in a problem makes the task harder to the students, something that does not occur when the source is implicit (Scaife & Heckler, 2010). A common interpretation for this fact is that students associate magnetic poles to electric charges and then they treat magnetic interaction as similar to electric interaction (Guisasola, Almudí & Zubimendi, 2004).

Even though many researchers consider models like those for explaining these didactical and psychological phenomena, for us, on the other hand, this is clear evidence that conceptualization depends on situations' parameters (Vergnaud, 2009; Vergnaud, 2013), videlicet, the source of magnetic field is different in both situations. Then, how can students tackle one and fail the other, which is blatantly similar to the first? Literature shows that learning difficulties are not uniform at all and maybe not subsumed by a single model, as Piaget would desire (Vergnaud, 2009). One fruitful way of answering these questions is by means of the construction of a conceptual field (Vergnaud, 1998). The one relative to the concept of magnetic field in static situations is inexistent in international literature.

To construct the referred conceptual field, it is necessary in first place to classify situations that compose it, based in the specific content to be used in problem-solving processes, namely, operatory invariants (Vergnaud, 2013; Vergnaud, 2009; Vergnaud, 1998). Following, one should analyze how students adapt their schemes to these situations. We name these classifications as epistemological and psychological, respectively, once the first one studies the content of knowledge available for being learned and the second one explores concepts actually learned by students. Pantoja and Moreira (2019) and Pantoja (2021) built epistemological classifications for electrostatic and for electrodynamics and, in this paper, we defend an epistemological classification we understand is filling this gap in the context of magnetostatic.

Thus, we propose the following research question: "how can situations involving the concept of magnetic field in static cases can be classified taking into account the theory of conceptual fields and the epistemological interpretation of the three facets?". Our goal is to design an epistemological classification of situations associated to the concept of magnetic field in the context of magnetostatics. Following, we discuss the theory of conceptual fields, the theoretical framework of this research.

THEORETICAL FRAMEWORK

The theory of conceptual fields is a complex cognitivist theory aiming at describing conceptualization processes, in other words, the establishment of reference to reality through representation (Vergnaud, 2013). Conceptual learning and cognitive development are complexly connected in conceptualization. Therefore, in this perspective, conceptualization is the cornerstone of cognition (Vergnaud, 2009).

In the theory of conceptual fields, concepts are defined by three sets: situations (S); signified (I); and signifiers (R). The inclusion of situations in the definition of concept makes itself necessary, once they compose the referential feature of the last one, the connection between reality and representation. Situations are tasks that make concepts useful and meaningful and, moreover, give them sense (Vergnaud, 2009). Operational invariants constitute the meaning of a concept, make it operational and integrate its meaning, then, can be understood as the signified of the concept. Representations (linguistic, symbolic and mathematic) permit representing operatory invariants and situations (Vergnaud, 2013), that is to say, they are systems of signifiers.

Knowledge can be, then, understood as a conceptual field, a set of situations, operational invariants, representations, thought operations and conceptual relations tied one to another and possibly intricated along learning and development process Vergnaud (2013). Scientific knowledge, specially, can be understood as a conceptual field, once there are classes of situations whose concepts are strongly related, while there are others that can be studied separately (Vergnaud, 1983). These divisions of conceptual fields are artificial because they involve arbitrary selection of concepts, but they are important, once we cannot study everything at a time and they end up having relevant pedagogic implications (Vergnaud, 2009).

A possible example is the difference between the conceptual fields of electrostatics and magnetostatics. The concept of electric current certainly presents a filiation between these two domains, however, the difficulties faced by students seem not be the same in these two conceptual fields. To state that, it is sufficient to remember that the concepts of electric field and magnetic field are strikingly different in static situations. The first one is parallel to the electric force and has nothing to do with the last one. On the other hand, the magnetic field is perpendicular to the magnetic force and has very little in common with electric field. Similarities include a few relevant features, like the relation between electric field and electric current density for ohmic media, which are insufficient to address these two conceptual fields as totally interdependent.

Using the notion of conceptual field implies admitting that learning difficulties depend on specific content, which makes impossible to subsume them to general logic operations, as Piaget wanted so. The theory of conceptual fields brings the notion of operational invariants, that can be understood as implicit or explicit conceptual basis underlying conceptualization, to consider the specificity of the content to be learned (Vergnaud, 2013). They are divided in concepts-in-action and theorems-in-action: the first ones are categories taken as pertinent about reality, while the second ones are understood as propositions regarded as true about reality. Operational invariants are attached to signified and signifiers but are not summarized as these two instances (Vergnaud, 2009).

Conceptualization includes reference to both objects and situations, nevertheless, reference to objects is clearer in explicit and verbalized forms of knowledge (predicative), while reference to situations is more evident in enactive forms of knowledge (operational). It is important to remember that conceptualization depends both on human activity and language (Vergnaud, 2013). Then, disregarding students' operational form of knowledge in evaluation of learning or in planning original didactic strategies is a wrong way out to the problem of conceptualization because great part of it is developed in action (Vergnaud, 2009).

The concept of scheme is used in the theory of conceptual fields to conceptualize human activity. This concept can be defined in four complementary forms (Vergnaud, 2013) as:

- a dynamical functional totality,
- an invariant organization of action for a certain class of situations,
- a structure composed of goals, rules of action (*if… then* like rules), operational invariants and possibilities of inference,
- A function that has its input values in a temporal space of n dimensions and its output values in a temporal space of n' dimensions, where n and n' are very large.

Situations are originally defined as tasks and conceptualization depend on its' parameters (Vergnaud, 1982). However, this perspective is excessively general and ends up by not clarifying the procedures used for classifying these situations. For example, it is hard to analyze how the situation depends on its parameters if they are not well defined. We propose to classify situations by using a typology, based on notions of *objects*, *variables* and *unknowns*, which was built in prior works (Pantoja & Moreira, 2019). We shall now define the meaning of such terms.

Objects are material entities related in the problem, that is, they have a mediate correspondence relation with reality (Bunge, 2011). We cite as examples a wire, a current conductor, a magnet, a solenoid conducting electric current and even a spinning conductor sphere. *Variables* are given or inferred information necessary for mastering the situation. For instance, the value of electric current produced by a wire, the magnetization of a magnet, the coil density in a solenoid or even the angular velocity of a charged sphere. *Unknowns* are elements to be discovered by the subject, by manipulation of variables, that relate the objects presented in the problem. For example, the magnetic field produced by the four mentioned systems. The attribution of the status of unknown or variable depends on what is asked in the situation (Pantoja & Moreira, 2019; Pantoja, 2021).

Structured problems include well-defined relations associating variables so that unknowns can be univocally found. Alternatively, when the variables are not structured so that a sole solution can be found or this one does depend on the adopted model, the problem can be considered ill-structured. For the latter, the parameters can be approached in distinct ways, once not all are actual data and some of them may be inferred, estimated, or regarded impertinent by subjects (Pantoja & Moreira, 2019; Pantoja, 2021).

Objects are always pertinent or not to a given situation, because it is always possible to consider them or not in the problem. Then, reference to objects is established by means of concepts-in-action, which possess characteristics of pertinence. Variables are double, once they can be arguments, which are deemed pertinent or not, as much as propositional functions, which can assume truth or falsehood values (Pantoja, 2021). Thus, reference to variables can be established by means of both concepts-in-action and theorems-in-action. In this perspective, solving a problem is finding a set of theorems-in-action and concepts-in-action describing one or more unknowns (Pantoja & Moreira, 2019; Pantoja, 2021).

We can set an example in electromagnetic theory. Electrons, for instance, are pertinent or not, once there is no sense in saying electrons are false or true, because they classify pertinent or impertinent real elements. By the other hand, stating the electron has electric charge equal to $1{,}6 \times 10^{-19} C$, which fits the description of a variable, includes truth or falsehood values. This statement is true from the scientific point of view. This is the basic difference between the concepts of theorem-in-action and concepts-in-action that is reflected in situations' parameters (Pantoja & Moreira, 2019; Vergnaud, 2013; Pantoja, 2021).

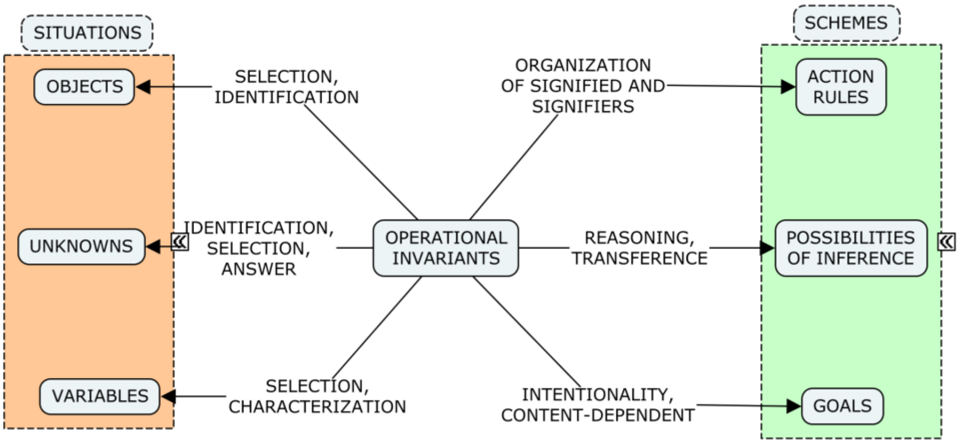

Figure 1 – Theoretical model for interaction between scheme and situation (Pantoja, 2021)

The theory of conceptual field considers that processes of learning and development occur when students adapt their prior schemes to new situations. These interaction between scheme and situation, understood as adaptation, occurs by means of the operational invariants, because they compose the explicit or implicit basis allowing the development of thought operations that relate objects, variables, and unknowns, which permits the connection between operational and predicative forms of knowledge while an individual tackles a conceptual field. Mastering of situations can be understood as adaptation of schemes to them and this process requires the creation, transformation or learning of new operational invariants. This just happens when the subjects tackle a situation in their zone of proximal development (Vergnaud, 2013).

In this article we focus on characterizing the left side of the diagram posed in figure 1, once we analyze situations. The complete study of the scheme-situation interaction requires, at first, the classification of situations and then the study of students' adaptation of their schemes to situations. For a complete characterization of a conceptual field, we must analyze how this interaction occurs for a great number of students of different cultures and educational levels, through pedagogical experiments, however it is beyond the scope of this article. It is impossible for a sole researcher to describe completely a conceptual field because of the conditions priorly described, then collaboration is fundamental for enlightening this issue.

We took the works on addictive and multiplicative structures due to Vergnaud (1982) as guidelines for building a proposal for the epistemological classification of tasks involving the concept of magnetic field in stationary cases. We describe the steps of this process in the next section.

THEORETICAL MODEL FOR CLASSIFICATION OF TASKS

The theoretical model for epistemological classification of tasks took into account central concepts of the conceptual field of the concept of magnetic field (in the context of magnetostatics). As long as no situation can be mastered with just one concept (Vergnaud, 1982), any situation in magnetostatics must associate at least two concepts of the conceptual field. Inasmuch as there are different forms of representing relations among concepts, for instance, natural language, symbols, images and even gestures, we chose to present the conceptual relations[1] in figure 1 by means of a concept map.

Magnetostatic considers slow and uniform motion of electric charge carriers. To approach high speed and accelerate motion requires regarding the retarding time and time effects associated to electromagnetic fields (Resnick, Halliday & Krane., 2006). Therefore, it would be essential to determine the Liènard-Wiechert due to an electric charge in generalized motion, something typical from electrodynamics (Jackson, 1999). Thus, the referred condition was left out of this analysis, but can be found in Pantoja (2021).

The Liènard-Wiechert field is "reduced" to the magnetic field due to a point charge in slow motion when the "adequate approximations" are done, namely, considering low speed as equivalent to the order of magnitude of the drift velocity in conducting wires. This implies linguistic operations "translating" two incommensurable epistemological fields[2] (Kuhn, 1997). An analogical epistemological shift is evidenced for

---

[1] We restricted the description to vacum to save space.

[2] Two paradigms are incommensurable when they cannot be translated one another, unless there is introduction of new elements of language in one of them (Kuhn, 1997).

magnetostatics and electrostatics (Pantoja & Moreira, 2019). We bring this discussion to justify why it is not possible to make a general analysis from the conceptual field of electrodynamics and simply specify the case of magnetostatics as the one in which magnetic fields do not vary in time (Pantoja, 2021). We make it in a separate way, once these two domains are epistemologically incommensurable (Kuhn, 1997).

For an epistemological analysis of the situations in a conceptual field it is necessary to characterize the manner by which the contents are structured in it. This includes describing the thought operations that are necessary to master certain class of situations, the meanings and representations attached to the situations, the numerical calculus that describe them and the possible relational calculus to be carried out in problem-solving (Vergnaud, 1982). In other words, it brings out the characterization of scientific accepted forms of interaction of scheme and situation, because numerical and relational calculus establish the connection between situations' parameters and thought operations allowed to be used to solve these tasks (Vergnaud, 1982).

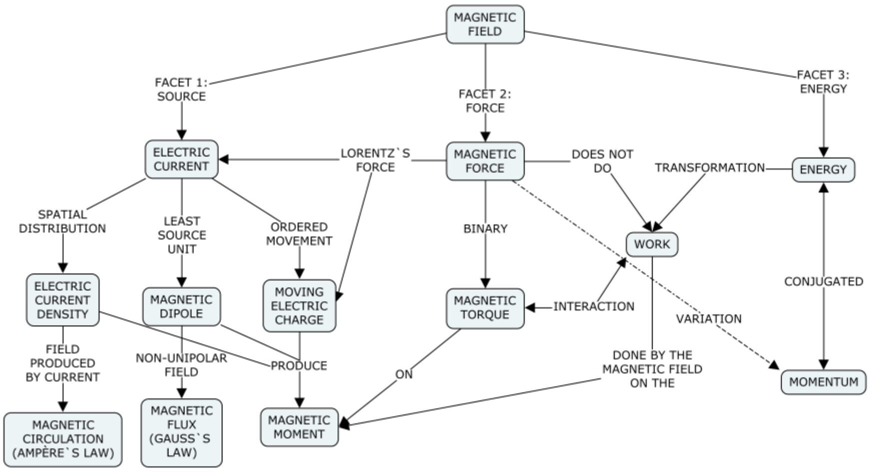

Figure 2 – Concept map with the main relations among the main concepts of the conceptual field of the concept of magnetic field in the context of magnetostatics.

We shall now distinguish the notions of numerical and relational calculus: the first one involves operational relations necessary for mastering a situation, while the second one calls for the thought operations needed to deal with relations in the situation (Vergnaud, 1982). It is important to highlight that relational and numerical calculus also involve thought operations of conceptual, qualitative, and typological manipulation, not just solely mathematical, quantitative, and topological operations. To explain why a "broken magnet" does not have "separate poles" involves numerical and relational calculus that involve the concept of magnetic field, for example. To make an example, we suppose two situations that require a sum as numerical calculus:

Situation 1: "Peter has 8 marbles in his right-hand pocket and 6 in his left-hand pocket. How many marbles does he have?"

Situation 2: "Peter has lost 6 marbles after playing a game and now has 8 marbles. How many marbles he had before playing?"

Vergnaud (1982) says both situations require summing 8 and 6 (numerical calculus), however they demand different thought operations that bring together different parameters (relational calculus). The first situation is classified as one in which there is a composition of measures, while the second one calls for a transformation between two measures.

Whilst in the first case, the concepts-in-action of part (6 and 8) and whole (14) are demanded to carry out the relational calculus, in the second one, the concepts-in-action of final state (8), transformation (-6), inverse of the transformation ($+6$) and initial state must be used, implicitly or explicitly, for solving the problem. Vergnaud (1982) states the second task is harder than the first one, because it requires greater mobilization of operational invariants for mastering the situation. Therefore, children that solve situation 1 for the first time can last one or two years more to master the situation 2, which demands the same numerical calculus (Vergnaud, 1982).

We, then, classified the situations based on a two-kind classification: primary (C_P) and secondary (C_S). Each C_P has the same thought operations necessary for its domain but can assume various groups of parameters. Each C_S needs the same thought operations and, moreover, share the same group of available parameters, that is, objects, variables and unknowns (Pantoja & Moreira, 2019; Pantoja, 2021). Thus, the problems of a C_P require the same numerical calculus to be mastered, whiles the tasks of a C_S demand both similar relational and numerical calculus.

On the other hand, thought operations were also classified in two types: general (h_g) and specific (h_e). The h_g are wider forms of thought that differentiate the C_P, that is, the different h_g are linked to distinct numerical calculus. The h_e are attached to the same numerical calculus, but to different relational calculus specifying how h_g are applied to the C_S. Both types of thought operations are related to the operational invariants and to representations, which implies in dependance on specific content of magnetostatics (Pantoja & Moreira, 2019; Pantoja, 2021). We do not want, by any means, to reduce any of the h_g or h_e to general logic thought operations, like Piaget wanted so (Vergnaud, 2013). Afterall, they depend on the implicit or explicit conceptual basis of the concepts.

The analysis was, then carried out by means of the following steps:

1) Determination of the parameters related in the situation,
2) Characterization of the set of the necessary h_g to solve the task, by means of the determination of the numerical calculus that brings the closure of the problem,
3) Determination of the C_P which belongs the situation,
4) Identification of the set of necessary h_e to solve the task, by means of the determination of the relational calculus that comes up with the solution providing the arrangement of the parameters of the specific task,
5) Establishment of the C_S in which the problem can be framed.

In the following section, we show how we established our classification based on these notions and give some examples of it.

RESULTS

Based on numerical calculi demanded for solving situations, we built four C_P and we refer to them using capital Greek letters. They require the *description of magnetic interactions* (Ψ), *analogic symbolization of the magnetic field* (Λ), *non-analogic symbolization of the magnetic field* (Ω) and *calculation of magnetic fields* (X).

The first C_P involves descriptions of momentum and energy changes among electrically charged moving objects. The second and the third ones call for a qualitative characterization of the field by means of analogic and non-analogic signifiers, respectively. Yet, the last one demands as numerical calculus the quantitative description of the magnetic field using functions.

In each of these C_P there are four different C_S, each of which share the same class of parameters, which imply in the existence of a myriad of distinct relational calculus needed to solve them. The C_S with a subscript A includes as objects moving point charges, the ones with subscript B involve situations in which there are sets of moving point charges, the C_S with subscript C are characterized by being associated to continuous and known distributions of electric currents and, by the other hand, the ones with subscript D entail situations with continuous and unknown distributions of electric charges. Chart 1 synthetizes the possible parameters attributed to distinct C_S.

Chart 1 – Comparison among parameters of each secondary class of situations.

C_S	**Parameters**
A	Moving point electric charge, electric charge, point position, source position, test charge position, velocity, distance between source and point, magnetic moment, magnetic field, amperian curve, gaussian surface, geometrical features related to the field, algebric-analytical features related to the field, magnetic flux, magnetic circulation, kinetic energy, linear momentum, angular momentum, acceleration, path, mass
B	All possible parameters for A, plus discrete and known distribution of moving point electric charges replacing an individual moving electric charge, resultant magnetic force (obtained by vector summation), resultant magnetic field (obtained by vector summation)
C	All possible parameters for B, plus continuous and known distribution of moving electric charges replacing discrete and known distributions of moving point electric charges, electric current density, electric current, magnetization and magnetic induction field, resultant magnetic force (obtained by vector integration), resultant magnetic field (obtained by vector summation)
D	All possible parameters for C, plus continuous and unknown distribution of moving electric charges replacing continuous and known distributions of moving electric charges, and boundary conditions for the magnetic field

We shall now discuss the specific h_g and h_e of each class of situations and bring examples, from two didactical books, chosen by convenience, which are representative of teaching materials used in Physics University courses around the world. These books are due to Resnick, Halliday & Krane (2006) and Jackson (1999).

Description of magnetic interactions

The common numerical calculus in every situation of the $\mathrm{C_P}$ for describing magnetic interactions (Ψ) is the determination of energy or momentum changes between test charges and a magnetic field produced by some source of electric current. Therefore, for solving problems of this kind it is necessary to use concepts as magnetic field, magnetic force, magnetic field momentum, magnetic field energy and magnetic torque. They involve four $\mathrm{h_g}$ for their resolution, namely:

i. *Recognition of magnetic field sources,*

ii. *Recognition of the interaction as magnetic in its' nature,*

iii. *Operational application of a magnetic interaction law,*

iv. *Quantitative or qualitative description of time evolution of the dynamical state of the interacting moving test charges.*

Recognition of magnetic field sources (i) requires two $\mathrm{h_e}$, to be specific, *the identification of magnetic field sources*, that calls for the enumeration of objects in the problem, and *characterization of magnetic field sources*, which refers to the description of variables attached to these objects. This $\mathrm{h_g}$ will be applied for all $\mathrm{C_P}$, once for all of them it is necessary to recognize sources of magnetic field, what shows how relevant the source facet is important for the electromagnetic theory (Nousianen & Koponen, 2017).

Recognition of the interaction as magnetic in its' nature (ii) demands three $\mathrm{h_e}$, namely, *description of the relation between magnetic force and magnetic field*, which involves the differentiation (or not) of these concepts from the operational point of view, *characterization of the ontological nature of the magnetic field*, that includes the philosophical interpretation of the magnetic field, and *description of the role of the field in magnetic interaction*, which aims at the conceptual description of the role of the magnetic field in interactions[3].

The operational application of a magnetic interaction law (iii) presupposes six $\mathrm{h_e}$: *operational application of magnetic force law*, that considers the operational, qualitative or quantitative, calculus of magnetic forces; *localization of energy in magnetic systems* and *localization of momentum in magnetic systems*, which relate to the description of where is the energy (in the field, in particles, both); *description of energy changes in magnetic systems* and *description of momentum changes*, that refer to characterization of the magnetic interaction by means of energy and momentum (by the field, directly by charges); and *application of the superposition principle*, which includes the conceptual use of the referred principle.

Last, for the quantitative or qualitative description of time evolution of the dynamical state of the interacting moving test charges (iv), two $\mathrm{h_e}$ are necessary: *schematization of the net force* and *description of the test objects movement*. These actions depend on knowledge from the conceptual field of classical mechanics, especially dynamics. The research program due to Hendrik Lorentz allows the establishment of the relation between classical dynamics (action at distance) and electromagnetic field theories (field scholars).

We, then, identified four possible $\mathrm{C_S}$ associated to this $\mathrm{C_P}$, and they require different relational calculus for their accomplishment. The following classes are:

- Ψ_A – interaction between two moving point electric charges,
- Ψ_B – interaction between a moving point charge and various moving point electric charges,
- Ψ_C – interaction between known continuous electric current distributions and one or more moving point electric charges distributed continuously or discretely,
- Ψ_D – interaction between unknown continuous electric current distributions and one or more moving point electric charges distributed continuously or discretely.

The first class of situations Ψ_A demands forms of relational calculus including as objects one moving electric point charge as source of magnetic fields and one moving point test electric charge. We can assume as variables the magnitude and direction of the force, velocities, distances, electric charge values, as long as derived quantities as, for example, kinetic energy, angular momentum and linear momentum. The unknowns depend on the form the question is posed – in structured problems, there is just one value for the unknown, whilst in ill-structured problems, there is a possibility of diverse values for that one or there can be more than one variable. Anyway, it is essential to adopt an unknown of interest. We present a structured problem that can be understood as a Ψ_A situation:

> Two positive point charges, $\mathrm{q_1}$ and $\mathrm{q_2}$, are moving to the right with the same velocity and are placed along a line which is perpendicular to their velocity ($\mathrm{q_1}$ is above $\mathrm{q_2}$). Which is the direction of the force on the charge $\mathrm{q_1}$ due to the magnetic field produced by $\mathrm{q_2}$? Adapted from Resnick, Halliday e Krane (2006)

The objects in this situation are two moving electric charges (1 e 2). The variables are their velocities, their values of electric charge, their relative positions with respect to an arbitrarily chosen reference system, the distance between the particles, and the magnetic field values produced by them in different points of the space. The unknown is the direction of the force (which implies in finding the acceleration).

Students must evoke the concepts of magnetic field and magnetic forces to relate the presented variables. The $\mathrm{h_e}$ of description of the relation between magnetic force and magnetic field and operational application of the magnetic force law, become more relevant, because the situation calls for relating the magnetic force and field by means of the use of an operational law of force. The last $\mathrm{h_g}$ (iv), requires actions from the conceptual field of mechanics for the direction of the force to be determined. The other thought

---

[3] The field is generally deemed a mathematical tool, a real entity or even inexistent (Guisasola, Almudí & Zubimendi, 2004).

operations are also important, but there is a great probability that they are implicitly evoked in students' answers once they were not explicitly demanded to address them.

The class of situations Ψ_B demands relational calculus considering a set of moving point electric charges and a moving point test electric charge. There is a wider range of possibilities for variables in comparison to the class Ψ_A, what implies in more complex relational calculus and thought operations. This class of situations entails two new kinds of variables related to the superposition principle, namely, the net magnetic field and the net magnetic force.

The same didactic limitations for Ψ_A are satisfied for Ψ_B. Therefore, using the model of a point electric charge in slow motion is not reasonable if one aims at modelling real physical systems, because it is necessary to discuss electromagnetic induction phenomena, the existence of electromagnetic waves, the computation of the magnetic field in the retarded time and the limitation to low-speed motion, for which there is low possibility of mathematical and physical demonstration. However, these models are valid if the goal is conceptual and didactic discussion, once they allow the teacher to relate the concepts of electric current, movement, drift velocity, magnetic field, magnetic force, as well as understanding the conducting wire as a large set of moving point charges in slow-motion. We present an example of situations of this kind as follows:

> Three positive charges q_1, q_2 and q_3 move to the right with the same velocity and are placed along a line which is perpendicular to their velocity (q_1 is above q_2, which is above q_3). What is the direction of the net magnetic force on q_2? Adapted from Resnick, Halliday & Krane (2006)

In this situation we have as new parameters the existence of one more moving point electric charge, an additional value of electric charge, one more velocity, a function describing the net magnetic field, one more position and two extra distances (from the two charges with relation to the third). In this task, we identified the relevance of the thought operation associated to the application of the superposition principle, which implies different relational calculus in comparison to Ψ_A. This principle did not apply to the latter and is used in its' discrete form in the case Ψ_B. The inclusion of this thought operation can turn the problem more complicated for beginners, once superposition principle is related to a set of physical and mathematical concepts in a non-trivial way (Rainson & Viennot, 1999).

The class Ψ_C calls for the concept of electric current density as novelty, that is to say, continuous distributions of moving electric charges are admitted as objects. These distributions are supposed to be known and fit the idea of distribution of electric current infinitesimals. This implies that the principle of superposition must be widened to entail continuous variations of electric charge by unit of volume, which brings the mathematical operation of integration to the problem. Then, forces and magnetic fields are computed with the aid of this concept.

Students often face huge difficulties of recognizing sources of magnetic field (Guisasola, Almudí & Zubimendi, 2004), what is compatible with the belief many of them sustain those sources of magnetic fields are intrinsic magnetic substances (Brandamante & Viennot, 2007). We see it is possible to approach these hindrances by means of a conceptual relation between discrete and continuous conceptual models. This connection can be established by introducing the continuous distributions as constituted by an infinity of point objects, that is, infinitesimals. Guisasola and colleagues call Amperian model, the one explaining the relation between magnets – macroscopic – and current coils – microscopic (Guisasola, Almudí & Zubimendi, 2003).

It is not uncommon that students employ different modes of conceptualization when the source of magnetic field is changed (Pantoja & Moreira, 2019b). There is empirical evidence showing that students' explanations suffer a wide variation if the source of magnetic field is implicit or explicitly taken as a magnet (Scaife & Heckler, 2010, Pantoja, 2015). It is possible that these two conditions may characterize two subclasses of Ψ_C in students' perceptions, but that could be understood by them as similar with the use of integrative concepts. This tertiary classification is beyond the scope of this work, because it demands the psychological classification of situations. We follow the discussion presenting an example of situation pertaining to the class Ψ_C:

> A long wire placed along the x axis is rigidly supported and conducts electric current $i = 96A$. A second wire which is directly above the first one and parallel to it conducts electric current $i_B = 23A$ and “weights” 0,073 N/m. At which distance, above the first wire, the second one must be held to be

> supported by magnetic repulsion? Adapted from Resnick, Halliday & Krane (2006)

In this situation, the objects are two long and horizontal wires conducting current. The variables are the two values of electric currents, the linear density of the wire that is supposed to be "floating", the value of local gravity acceleration, the magnetic field, and the position of the "center"[4] of the wires. The unknown is the value of the magnetic force needed to leave the system in rest ($\mathrm{a} = 0$ and $\mathrm{v} = 0$ are two more variables). The relation between electric current and electric charge in movement is implicit but can be worked to demark ruptures and filiations with respect to the class of situations Ψ_{B}.

The $\mathrm{h_e}$ for relating magnetic field and force, the operational application of a magnetic force law and for applying the superposition principle, are central for solving this kind of situation. The latter transforms the former ones, once it is computed as a continuous variable and produces an appliable model for the force between two long wires. The other $\mathrm{h_e}$[5], relative to the determination of the interaction law and to the description of the role of the magnetic field in interaction, are inclined to remain implicit, because there are epistemological and ontological features related to the magnetic field that are usually not discussed in the classroom, once science teachers have strong difficulties in mastering these topics of Nature of Science (El-Khalic & Lederman, 2000). To help students to make them explicit, it is possible to ask about energy balance, localization of electromagnetic energy, and how momentum changes are processed, for example.

The class Ψ_{D} involves problems in which the distribution of electric currents must be determined after finding the magnetic field. This determination is made by the use of differential equations for regions where there are no electric currents. Ampère's and magnetic Gauss's laws are taken as premise and one supposes the curl of the magnetic field is null. Laplace's equation is, then, obtained by means of this procedure and solving it requires setting the boundary conditions satisfied by the magnetic field. These ones require the normal magnetic field component to the surface be continuous, while the parallel component must be discontinuous and equaled to the free electric currents of the problem. It is possible to determine the magnetic force on a moving point electric charge from the combination of the Lorentz's force and the obtained magnetic field. The same scenario occurs when there are "magnetic materials" in the problem, although the same boundary conditions are imposed on magnetic induction[6] ($\vec{\mathrm{H}}$) and on the magnetic field ($\vec{\mathrm{B}}$). Here follows an example:

> Let be a sphere, of radius R, and magnetic permeability μ put in a region where there is a uniform magnetic field. Which equation of motion describes the movement of a point electric charge moving with velocity $\vec{v} = v_0\hat{k}$ located at distance d far from the center of the sphere? Adapted from Jackson (1999)

There are two objects, the sphere and the moving electric charge. The variables of the problem are the radius of the sphere, the permeability of the sphere, the uniform magnetic field, the net magnetic field, the absence of free electric currents in the sphere, the electric charge of the point object, its velocity, its distance to the center of the sphere, the magnetization of the sphere and its magnetic induction. The unknown is an equation describing the particle's position throughout time.

There is some sort of difference in relational calculus demanded to solve this situation and it is related to the superposition principle. After solving the differential equation, we already have the net magnetic scalar potential, whose gradient leads to the net magnetic field. In situations of the type Ψ_{C}, one applies the superposition principle by performing an integral, an explicit summation, which is not the case in situations Ψ_{D}. Ampère's and magnetic Gauss's law entail implicitly the superposition principle both in differential and integral forms, what is compatible with Ψ_{D} situations, while in Ψ_{C} tasks a direct integration must be evaluated. Therefore, the superposition principle does not come out explicitly, and this may increase difficulty in comprehending this idea that does not come up in the other classes of situations. Moreover, this class demands more complex concepts and mathematical calculus, and that is why it is usually approached in disciplines of the professional cycle of Physics Courses.

---

[4] Very long wires can be approached as infinite; therefore, they have no center.

[5] Characterizing ontologically the magnetic field, describing the role of the field in magnetic interaction, localizing and describing momentum and energy changes in magnetic systems.

6 We nominate magnetic field and magnetic induction the other way around to keep it compatible with the classification made in electrostatics (Pantoja & Moreira, 2019).

Analogic symbolization of magnetic fields and non-analogic symbolization of magnetic fields

In first place, it is important to stress why the name symbolization was chosen in place of the term representation. This was made this way because ambiguous terminology is usually adopted for these two terms. In this paper representation is understood as a dynamical cognitive process underlying conceptualization (Vergnaud, 1998). Symbolization, by the other hand, is comprehended as a process of using different semiotic systems to present objects and their properties in their absence (Vergnaud, 2013). Representation entails both operational and predicative forms of knowledge, while symbolization is clearly predicative, once it involves the process of using explicit symbols to express knowledge. Then, symbolization can be understood as the predicative counterpart of representation, however it depends on representation, which is a larger set including it.

The common numerical calculus to situations for which there must happen analogic symbolization of magnetic field (Λ) involves qualitative characterization of the magnetic field with the aid of symbols possessing structural analogy with the field, like draws and pictorial diagrams. On the other hand, the constant numerical calculus in situations requiring non-analogic symbolization of the magnetic field (Ω) include qualitative characterization of the field by means of symbols having arbitrary structural relation with the field, for instance, equations and words that do not describe visuospatial layouts. Then, situations in which must occur symbolization of the magnetic field call for five types of h_g for being solved, namely:

1. *Recognition of magnetic field sources,*
2. *Recognition of analogic (Λ) or non-analogic (Ω) features of the sources,*
3. *Localization of points in space,*
4. *Translation of the meaning of field equations as analogic (Λ) or non-analogic (Ω) features,*
5. *Association of vectors or scalars to points in space for establishing analogic (Λ) or non-analogic (Ω) symbolization.*

These types of situations seem to be similar, but the demands for different thought operations for building analogic representations and non-analogic representations are strongly different (Markman, 1999). Our chief motivation to classify them in distinct groups is based on the argument that some people prefer to deal with analogic representations while others feel more comfortable with the non-analogic ones, therefore, these competences must be different in a cognitive basis. To develop mediational acts to integrate these two instances is part of the role of the teacher (Vergnaud, 2013). Mathematically, we attached analogic symbolizations to the conceptual field of geometry (plain and spatial), because this one establishes a relation of analogic structure with what it aims to symbolize, and the non-analogic ones to the conceptual fields of algebra and analysis, once the latter ones are arbitrary representations that do not possess structural analogy with what they aim to represent (Nascimento, 2017).

After developing the recognition of magnetic field sources (i), it is necessary to take into account analogic aspects (Λ) and non-analogic (Ω) of the sources (ii). These h_g are related to ideas belonging to the conceptual fields of geometry and algebra-analysis, then the h_e will process accordingly to that. First, it is necessary to identify analogic (Λ) or non-analogic (Ω) features of the sources, by making reference to important parameters characterizing the sources as, for example, to indicate the cylindrical shape of a wire. Second, it is demanded to characterize analogic (Λ) or non-analogic (Ω) attributes of the source as, for instance, when one describes the wire's radius or height.

Magnitude and direction of magnetic fields depend on the localization of points in space (iii). Magnetic flux and magnetic circulation also differ accordingly to the different points chosen to be involved by a gaussian surface or an amperian curve. This h_g can be splitted in three other ones, to be specific, determination of positions of points in space, determination of positions of sources of magnetic field and determinations of distances between point and source.

It is necessary to establish the translation of the meaning of field equations into analogic (Λ) or non-analogic (Ω) features (iv). This h_g can be divided into two h_e, namely, physical interpretation of the magnetic flux and physical interpretation of magnetic circulation. Complete knowledge of magnetic fields calls for knowing both the flux (or the divergent) and the circulation (or the curl) of this vector field, which turns the existence of these two h_e essential, once they bear on the physical interpretation of two mathematical quantities. The signifiers linked with this interpretation can be both analogic and non-analogic.

The last stage is employing the h_g of association of vectors or scalars to points in space for establishing analogic (Λ) or non-analogic (Ω) representation (v). It can be ramified in three h_e, to be specific, namely, the interpretation of the relation between magnetic flux and magnetic field, interpretation of the relation between magnetic circulation and magnetic field and construction of the symbolization *per se* in analogic (Λ) or non-analogic (Ω) terms. It worth highlighting that magnetic field equations already carry with them the superposition principle implicitly.

We identified four C_S related to each of the C_P, based on their characteristic parameters:

- Λ_A or Ω_A – analogic or non-analogic symbolization of a magnetic field due to a moving point electric charge,
- Λ_B or Ω_B – analogic or non-analogic symbolization of a magnetic field due to a set of moving point charges,
- Λ_C or Ω_C – analogic or non-analogic symbolization of a magnetic field due to a continuous and known electric current distribution,
- Λ_D or Ω_D – analogic or non-analogic symbolization of a magnetic field due to a continuous and unknown electric current distribution.

Classes Λ_A and Ω_A require analogic and non-analogic symbolization of magnetic fields due to moving point charges. The possible object is a single moving point charge. We can enumerate values of electric charge, velocity and distance between charge and point of evaluation of the field as variables. Situations Λ_A can ask for construction of diagrams as unknown, while tasks Ω_A can call for an abstract relation between two physical quantities. We present two cases for comparison:

> A point charge is moving to the right along the positive direction of the x semiaxis. Draw the magnetic field lines representing the magnetic field produced by it in space. How is the shape modified if we increase the particle's velocity? What if we diminish it? And if we change direction? Adapted from Resnick, Halliday & Krane (2006).
>
> A point charge is moving to the right along the positive direction of the x semiaxis. How the magnetic field produced by it in space is coherent with Ampère's and magnetic Gauss's law? Adapted from Resnick, Halliday & Krane (2006).

Objects and variables are the same in both situations, namely, a moving point electric charge, points in space, electric charge, velocity, distances, positions, Amperian curve and gaussian surface. On the other hand, the unknowns are different. The first case calls for analogic representation and the second one requires establishing an abstract relation of coherence between the magnetic field and the laws supporting it. We take as example the solenoidal characteristic of the magnetic field – it is easier to approach it by means of geometrical forms in situations Λ (analogic), while is more convenient to highlight a relation between the non-unipolar character of the magnetic field and the magnetic flux in situations Ω. Grossly speaking, it is possible to say that if Λ situations focus on the "shape of the magnetic field", the Ω ones are driven to general relations among concepts.

Λ_B and Ω_B situations require analogic and non-analogic symbolization of the magnetic field due to a set of moving point electric-charges, respectively. The superposition principle is qualitatively included in this class of situations. Possible objects involve moving point electric charges and variables can be distances from charges to points where the field is determined, values of electric charges and the velocities of the point charges. We present two examples to sustain the differentiation between the numerical calculi needed to solve these two types of situations:

> Two points charges are moving to the right along the positive x-axis, with the same velocity, and distant d far from the other (one is above the other). Draw an arrow diagram of the net magnetic field produced in space. How does the shape of the diagram change if we increase the speed of the particles? What if we diminish it? And if we change direction? Adapted from Resnick, Halliday & Krane (2006)

> Two point charges are moving to the right along the positive x-axis, with the same velocity, and distant d far from the other (one is above the other). Suppose that one of the charges crosses the surface enclosed by an Amperian curve, the magnetic field in Ampère's law is solely due to it? Explain. Adapted from Resnick, Halliday & Krane (2006)

The same reasoning employed to the prior situations will be used for interpretation of these tasks. The superposition principle must be explicitly incorporated, in virtue of the set of moving point electric charges. This principle is already attached to Ampère's and magnetic Gauss's law, once both magnetic flux and magnetic circulation are determined by means of the net magnetic field. Introducing one more electric charge as object obviously implies new variables, namely, electric charge values, velocities, positions and distances, but the main difference is the inclusion of the superposition principle in the relational calculus in Λ_B and Ω_B. We suppose it is already clear that the unknowns in these two situations call for different numerical calculus and that is why they are found in distinct C_P.

Classes Λ_C and Ω_C call for analogic or non-analogic symbolization of the magnetic field due to continuous and know stationary electric current or dipole moment distributions. The differences with respect to the prior C_S are both in physics and mathematics, once now the concepts of electric current density and of magnetization[7] are presented a priori. Physically, two new variables besides these presented are involved, namely, distance between point and source, electric charge and velocity. Mathematically, the inclusion of integration operations, understood as summation on continuous values, is a novelty. We present two examples as follows:

> A horizontal long rigidly supported wire conducts an electric current $i = 96A$. Sketch the field line diagram of the magnetic field. Adapted from Resnick, Halliday & Krane (2006)
>
> A horizontal long rigidly supported wire conducts an electric current $i = 96A$. Is there any eligible gaussian surface through which there is a non-zero magnetic flux? Explain your reasoning. Adapted from Resnick, Halliday & Krane (2006)

Considering the infinitesimal elements of electric charge in ordered motion is a complicating factor in this C_S. Both situations have objects and variables in common, videlicet, a conducting wire (object), the electric current, the position of the wire with respect to the reference system and relative distances to the wire (variables). Nevertheless, the situation Λ_C requires analogic representation of the magnetic field, whilst the Ω_C situation calls for non-analogic relation among the concepts of magnetic flux, gaussian surface and magnetic field. The necessary h_g and, consequently, the h_e follow different cognitive paths, even though the objects and variables are the same, therefore, this feature highlights unknowns are also crucial to define the C_P which we are dealing with.

Classes Λ_D and Ω_D demand, respectively, the analogic symbolization of the magnetic field due to a continuous distribution of stationary electric currents or dipole moments which are unknown a priori. This may be the hardest of all the C_S relative to these two C_P, once they include all possible variables of the prior classes and the notion of redistribution of electric current or the magnetization as parameters. By the way, situations involving determination of the orientation of magnetic fields due to magnets fits in this kind of task. There is clearly rupture between scientific view and students' view because the latter tend to understand magnets are constituted of an especial kind of matter, which has "magnetic" properties (Brandamante & Viennot, 2007), while the former admit magnets have magnetization due to intrinsic magnetic moment, which consist essentially in a relation between electric charge and angular momentum, and this implies "magnetic materials" are formed of ordinary matter (Jackson, 1999). There is a slow path of conceptualization to follow until integrating these two incompatible notions once this seems to be epistemologically the most difficult type of situations to master and psychologically students' prior knowledge rest on common knowledge on magnets. The qualitative feature related to comprehending differential equations as mathematical structures describing quantities in a local form is a fundamental point to stress in the mastering process of this kind of situation. We present two examples, one of the Λ_D kind and the other of the Ω_D type, both we the same variables and objects and distinct unknowns:

---

[7] Volumetric variation rate of the quantity of dipole moments.

> Let be a sphere, of radius R, and magnetic permeability μ, placed in a region where there is a uniform magnetic field. How would you sketch the magnetic field line diagram for this system? Adapted from Jackson (1999)
>
> Let be a sphere, of radius R, and magnetic permeability μ, placed in a region where there is a uniform magnetic field. How is it possible to articulate the superposition principle to Ampère's and magnetic Gauss's law for justifying the applicability of them to the problem? Adapted from Jackson (1999)

As already highlighted, the differences to stress are in the unknowns. The Ω_D type allows the student to establish a fundamental relation on the understanding of Ampère's and magnetic Gauss's laws, that is to say, that the magnetic flux and circulation refer to the net magnetic field. By the other hand, the situation Λ_D kind calls for a structural correspondence relation with magnetic field direction. The unknown is, then, a functional analogic or non-analogic relation among variables and contains truth or falsehood value (a theorem-in-action). In the following section, we discuss the last C_P, which requires the calculation of magnetic fields.

Calculation of magnetic fields

The numerical calculus common to all situations in which one must calculate magnetic fields in space (X) involves determining a vector-valued function of several real variables (points in space). Therefore, from knowledge about the sources and boundary conditions of the magnetic field, it is necessary to find a function describing the magnetic field and then interpret it. Situations which call for calculation of magnetic field demand five h_g, namely:

i. Recognition of magnetic field sources,
ii. Recognition of mathematical features related to the source,
iii. Determination of distances between source of magnetic field and point of calculation,
iv. Execution of the calculus of the magnetic field using physical laws,
v. Interpretation of the resulting calculation.

Situations X require recruiting different concepts and thought operations in comparison to classes Ψ, Λ and Ω, thus, they are essentially different from the priorly presented classes of situations. The first three h_g are basically similar to the ones related to classes Λ and Ω, but the fourth and the fifth ones are determinant for distinguishing between X and the other two. Besides, the mathematical attributes relative to calculation of the field, at the same time, come from elements from algebra, analysis, and geometry as crucial for mastering the situation.

Execution of the calculus of the magnetic field using physical laws (iv) includes four h_e, videlicet: *application of an adequate physical law for calculation of the magnetic field*, which comes from the necessity of choosing a physical law for operationally determining the magnetic field; *computation of the magnetic field at different distances,* once it is necessary to evaluate the relation between field based on the distance between source and point; *determination of the direction of the magnetic field from data due to electric current*, because is essential to indicate the direction of the magnetic field by qualitative means; and *application of the superposition principle for calculation of the field*, once it is tied to the dependance on the superposition principle for finding the net magnetic field.

Interpretation of the resulting calculation (iv) comes through two h_e, namely, *contrast between the result obtained for the magnetic field and the magnetic Gauss's law*, once the magnetic field is necessarily non-monopolar, and *contrast obtained for the magnetic field and the Ampère's law*, because it must also be solenoidal and produced by electric currents. These thought operations are essential for mastering situations X.

We identified four classes of C_S for X, based on the characteristic parameters of this C_P and on the h_e need for mastering the situation:

- $\mathrm{X_A}$ – Calculation of the magnetic field due to a moving point electric charge,

- $\mathrm{X_B}$ – Calculation of the magnetic field due to a set of moving point electric charges,
- $\mathrm{X_C}$ – Calculation of the magnetic field due to a continuous and known distribution of point electric charges in ordered motion (electric current),
- $\mathrm{X_D}$ – Calculation of the magnetic field due to a continuous and unknown distribution of point electric charges in ordered motion (electric current).

Situations X_A are tasks requiring the calculation of magnetic fields due to a single point electric charge in slow motion. The only possibility of object is the moving point electric charge. As variables, one can arrange combinations of distance between source and point of calculation, magnetic field, velocity of the moving point charge and values of electric charge. Unknowns depend on how the problem is proposed. Here follows an example.

> A proton $(q = 1{,}6 \times 10^{-19} C)$ moves to the right, along the positive x semiaxis at a speed $v = 9 \times 10^3 m/s$. Which is the intensity of the magnetic field produced by a proton distant one meter far from it? Adapted from Resnick, Halliday & Krane (2006).

The object is clearly a moving proton. The variables are the values of electric charge, mass, velocity and distance from point to source. The unknown is the value of the magnetic field, once this is what must be found for the task to be solved. The equation for the magnetic field produced by a moving electric charge is widely known. It is evident that the h_g and h_e are different from the ones required for Λ and Ω, once they entail different numerical and relational calculus.

Situations X_B call for the calculation of magnetic fields produced by a discrete distribution of moving point electric charges. The superposition principle must be adopted as a sum, because there is a discrete distribution of electric charges. The variables are values of electric charge, distances from source to point of calculation and velocities of the moving point charges. We shall present an example:

> Two point electric charges, q_A and q_B, are moving to the right along the positive x semiaxis with the same speed v. Which vector describes the net magnetic field produced in any point of the space? Adapted from Resnick, Halliday & Krane (2006).

There are two main differences turning this class of situations harder than the priorly mentioned, X_A. The first of them is requiring both intensity and direction of the field, what calls for another unknown to the problem, and the second one is that superposition principle is evoked. The expression to be used in this specific problem is the “Biot-Savart law”[8] for electric charges in slow motion:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \vec{r}}{r^3}$$

Situations X_C require the calculation of the magnetic field due to continuous and known distribution of point electric charges in ordered motion. In other words, a stationary distribution of electric currents or magnetic dipole moments. The additional physical variables are the electric current density and magnetization. The mathematical structure of integration is fundamental for using Ampère’s law or Biot-Savart’s law. We present an example as follows:

> A horizontal long wire, rigidly supported, conducts an electric current $i = 96A$. Which are the magnitude and direction of the magnetic field due to the wire at a point P, distant $r = 0.5m$ far from the wire, along the positive x semiaxis? Adapted from Resnick, Halliday & Krane (2006)

There are both ruptures and filiations in this situation. The filiation is in the continuity of the field equation and the rupture is in considering the electric current as a continuous and know distribution of moving point charges. The unknown to be found is the magnetic field vector, but finding it requires certain steps to be given. For instance, considering a wire conducting neutral currents composed of various positive charges moving in a region filled with negative charges so that the total electric charge density be constant throughout time (Lemos, 1989). This leads to the “Biot-Savart’s law”, once we can understand an

---

[8] Biot and Savart have never spoken about the magnetic field, it was always about magnetic force.

infinitesimal electric charge element as $dq = idt$ and a path element described by it in time as $\vec{v}dt = d\vec{l}$. This reasoning evokes the operation of integration for solving situations like this one. Many professors seem to deem the operational calculus with integrals the complicating factor when one passes from X_B to X_C, but we admit that the feature demarking the epistemological frontier between these two classes of situations is the conceptual passage from discrete distributions to continuous distributions. Our argument involves the notion that establishing a filiation between continuity and discreteness, which bring a clear rupture, requires relating the continuous as a set of a great number of “tiny” discrete particles that occupy a volume which is both big enough, so it involves a considerable number of particles and sufficiently small that it is not finite. In other words, conceptualizing it involves clearly understanding the relation between microscopic and macroscopic (Guisasola, Almudí & Zubimendi, 2004), something certainly not easy including for a huge number of scientists.

Situations X_D demand the calculation of the magnetic field due to a continuous and unknown distribution of moving point electric charges (in slow motion). These distributions are known *a posteriori* from the boundary conditions imposed to the field, therefore, the notions of boundary condition of the magnetic field, differential forms of magnetic field laws and determination of the electric current density *a posteriori* are essential for solving this class of situations. We regard X_D as the epistemologically most difficult class of situations of the four listed, because it involves more elements and needs deeper and more detailed physical-mathematical knowledge than the essential for resolution of these problems. We set an example for illustration.

> Consider a sphere, of radius R, and permeability μ, placed in a region where there is a constant magnetic field. Which vector functions describe net magnetic induction, $\vec{H}$, and the net magnetic field $\vec{B}$ in space? Adapted from Jackson (1999)

Magnetic field and magnetic induction were brought out as unknowns so it would be possible to show the greater difficulty of this class of situations. In these tasks the students must approach the behavior of magnetic fields in cases in which there are magnetized objects. Moreover, superposition principle is implicit, and comprehension of this feature is turned harder by poor discussion usually made by physics didactic books. Determining the field before knowing its distribution and adapt it to boundary conditions may mobilize physical reasoning, but mathematically requires local analysis of how the field varies with distance. We finish this paper bringing the conclusions and discuss on pedagogical implications of this research.

CONCLUSIONS AND DISCUSSION

We built in this work a theoretical model for epistemological classification of situations involving the concept of magnetic field in magnetostatics by means of the theory of conceptual fields (Vergnaud, 2009). Four primary classes of situations (C_P) were built, namely, description of magnetic interactions (Ψ), analogic symbolization of magnetic fields (Λ), non-analogic symbolization of magnetic fields (Ω) and calculation of magnetic fields (X). Each of which has four secondary classes of situations (C_S) differing among them for having different groups of parameters that apply to them. This construction was made using as criteria the numerical and relational calculi required for solving each class of situations. While C_P are structured around general thought operations (h_g), C_S are organized in terms of specific thought operations (h_e) necessary to solve tasks they refer to.

A measure of the complexity of the C_S is the number of operational invariants in such of them. How did we arrive at this conclusion? We introduced various concepts inasmuch as we changed from one class to another, for instance, changing from X_A to X_C, and by assuming it is easier to deal with a minor number of operational invariants than with a greater number, we concluded that classes indexed by D of all C_P are more complex than the classes A, because the prior have much more possible parameters than the latter, for example, the concept of electric current density and the relations of superposition that do not exist in class A. In the presented example on additive structures, Vergnaud (1982) shows empirically that children can, in mean, deal better with compositions of two measures than with transformation between two measures, because there is a larger set of parameters for latter class of situations.

Which factors can we manipulate in order to change from one class to another? We use the same examples of the four C_P. We changed unknowns to evidence the variation among them and we kept objects and variables constant. By the other hand, we altered objects and variables to evidence the variation among C_S, besides keeping unknowns constant. Adjustments on situations parameters have great influence in required competences to solve the problems and in difficulties of mastering these tasks (Scaife & Heckler, 2010; Vergnaud, 1982). These recasts must provide conditions for gradually more complex learning processes.

It is important to indicate that the source facet is present in all C_P, while the force and energy facets are emphasized by the description of magnetic interactions (Ψ). Does this indicate some kind of asymmetry? We do not understand this way, once the concept of magnetic field is central in all of them. Then, the energy and force facets are also in all classes of situations when one mentions there is energy and momentum in the field. Force can be understood as momentum transfer and work as energy transfer (or transformation), both mediated by the magnetic field, which implies the three facets are included in the four C_P (Nousianen & Koponen, 2017). We present this notion as an interesting frontier for distinguishing between force and field, which, by the way, is one of the great difficulties faced by students (Guisasola, Almudí, & Zubimendi, 2004).

This research offers three contributions for Science Teaching Research. The first one is offering a theoretical framework for analyzing students' progression in the conceptual field of the concept of magnetic field in magnetostatics, something still inexistent in research literature. The second one includes the proposition of a conceptual structure for building situations based on predicative and operational competences for understanding this concept, which can enlighten the planning process of teachers' mediational acts during didactical process (Vergnaud, 2013). The last one includes a practical example of epistemological classification that can be reproduced in other areas of Physics in other scientific fields as, for example, Quantum Mechanics.

Psychological classification of situations is a deliberately point which we did not approach in this work, once the required extension for its discussion would turn this investigation unnecessarily long. One example of the need of the classification of this kind is related to the evidence that the omission of the magnetic field source and the expression of the field lines facilitates learning of the concept of magnetic force (Scaife & Heckler, 2010). This data is derived from a pedagogical experience for testing the degree of difficulty of the problems when proposed to the students. The determination of tertiary classes of situations (C_T) needs the conduction of empirical studies on students' conceptualizations and that is why it can be understood as a psychological classification. A good example is analyzing if it is easier for the students to determine the magnetic field due to rotating charged sphere or due to a very long straight wire conducting electric current. It is fundamental that one analyses carefully the elaboration of subclasses taking into account the modes of mobilization of operational invariants. The degree of difficulty must be the inverse of the success rate obtained by students. It is just possible to know if difficulties are local or universal with the aid of researchers from different sociocultural contexts.

A reflection on the form by which textbooks propose situations to the students is pertinent for finishing this didactical analysis. We classified the tasks on magnetic field on the textbook due to Resnick, Halliday & Krane (2006) for understanding which C_P was more often recalled by the authors. We identified that 64 are compatible with X situations, what can be an indicative of sustaining an instrumentalist view of the authors (Bunge, 2011), once the emphasis in tasks of calculation of magnetic fields and little discussion on the role of the magnetic field on magnetic interaction can suggest to the student that the only function of this construct is being a mathematical tool (Uhden, Karam, Pietrocola & Pospiech, 2012). This conception was already evidenced in other consolidated research discussing the conceptual approaches for the concept of magnetic field (Krapas & da Silva, 2008; Pocovi & Finley, 2003). However, conceptual, and phenomenological approach for teaching the concept of magnetic field is the most recommended for providing better conditions for a comprehensive learning of this concept (Zuza *et al.*, 2018).

REFERENCES

Abd-El-Khalick, F., & Lederman, N.G. (2000). Improving science teachers' conceptions of nature of science: A critical review of the literature. *International Journal of Science Education*, *22*(7), 665–701. https://doi.org/10.1080/09500690050044044

Bollen, L., Van kampen, P., Baily, C., & de Cock, M (2016). Qualitative investigation into students' use of divergence and curl in electromagnetism, *Physical Review Physics Education Research*, *12*, 020134. https://doi.org/10.1103/PhysRevPhysEducRes.12.020134

Bradamante, F., & Viennot, L. (2007) Mapping Gravitational and Magnetic Fields with Children 9–11: Relevance, difficulties and prospects. *International Journal of Science Education*, *29*(3), 349–372. https://doi.org/10.1080/09500690600718245

Bunge, M. (2011) *Caçando a realidade: a luta pelo realismo*. São Paulo, SP: Perspectiva.

Campos E., Zavala, G., Zuza, K., & Guisasola, J. (2020) Students' understanding of the concept of the electric field through conversions of multiple representations. *Physical Review Physics Education Research*, *16*, 010135 https://doi.org/10.1103/PhysRevPhysEducRes.16.010135

Galili, I. (1995) Mechanics background influences students' conceptions in electromagnetism. *International Journal of Science Education*, *17*(3), 371-387. https://doi.org/10.1080/0950069950170308

Guisasola, J., Almudí, J. M., Salinas, J. Zuza, K., & Ceberio, M. (2008) The Gauss and Ampere laws: different laws but similar difficulties for students learning. *European Journal of Physics*, *29*, 1005-1016. http://dx.doi.org/10.1088/0143-0807/29/5/013

Guisasola, J., Almudí, J. M., & Zubimendi, J. L. (2003) Dificultades de aprendizaje de los estudiantes universitarios en la teoria del Campo Magnético y elección de los objetivos de enseñanza. *Enseñanza de las Ciencias*, *21*(1), 79-94. https://doi.org/10.1590/S1806-11172010000100011

Guisasola, J., Almudí, J., & Zubimendi, J. (2004) Difficulties in Learning the Introductory Magnetic Field Theory in the First Years of University. *Science Education*, *88* (3), 443–464. https://doi.org/10.1002/sce.10119

Jackson, J. D. (1999). *Classical Electrodynamics*. New York, United States of America: John Wiley & Sons.

Krapas, S., & Silva, M. (2008) O conceito de campo: polissemia nos manuais, significados na física do passado e da atualidade. *Ciência & Educação (Bauru)*, *14*(1), 15-53. https://doi.org/10.1590/S1516-73132008000100002

Kuhn, T. S. (1997) *A estrutura das revoluções científicas*. São Paulo, SP: Perspectiva.

Lemos, N. (1974) Energia magnetostática: uma dedução simples e geral. *Revista Brasileira de Ensino de Física*. *11*(1), 24-29. Recuperado de http://www.sbfisica.org.br/rbef/pdf/vol11a03.pdf

Markman, A. (1999) *Knowledge representation*. Mahwah, United States of America: Lawrence Erlbaum Associates.

Mbonyiryivuze A., Yadav, L. L., & Amadalo, M. M. (2020) Students' conceptual understanding of electricity and magnetism and its implications: a review. *African Journal of Educational Studies in Mathematics and Sciences*, *15* (2), 55-67. https://doi.org/10.4314/ajesms.v15i2.5

Nascimento, E. M. (2017) *Integração entre álgebra e geometria no ensino da Matemática*. (Dissertação de mestrado). Mestrado Profissional em Matemática. Universidade Federal do Viçosa, Viçosa, MG. Recuperado de https://www.locus.ufv.br/handle/123456789/11710

Nousianen, M., & Koponen, I. (2017) Pre-service physics teachers' content knowledge of electric and magnetic field concepts: Conceptual facets and their balance. *European Journal of Science and Mathematics Education*, *5*(1), 74-20. https://doi.org/10.30935/scimath/9499

Pantoja, G. C. (2015). *Unidades de ensino potencialmente significativas em teoria eletromagnética: influências na aprendizagem de alunos de graduação e uma proposta inicial de um campo conceitual para o conceito de Campo Eletromagnético*. (Tese de Doutorado). Programa de Pós-Graduação em Ensino de Física. Universidade Federal do Rio Grande do Sul, Porto Alegre, RS.

Pantoja, G. C., & Moreira, M. A. (2019a) Classificação de problemas em eletrostática: uma análise epistemológica rumo à construção de um campo conceitual para o conceito de campo eletrostático. *Latin American Journal of Physics Education*, *13*(4), 4304. Recuperado de http://www.lajpe.org/dec19/13_4_04.pdf

Pantoja, G. C., & Moreira, M. A. (2019b) Investigando a implementação de uma Unidade de Ensino Potencialmente Significativa sobre o conceito de Campo Magnético em disciplinas de Física Geral.

Revista Electronica de Investigación en Educación en Ciencias, *14*(2), 1-16. Recuperado de http://www.scielo.org.ar/pdf/reiec/v14n2/v14n2a01.pdf

Pantoja, G. C. (2021). Campos Conceituais e Indução Eletromagnética: Classificação de Problemas em Eletrodinâmica. *Revista Brasileira de Pesquisa em Educação em Ciências*, e24370, 1-33. https://doi.org/10.28976/1984-2686rbpec2021u441473

Pocovi, M. C, & Finley, F. (2003) Historical Evolution of the Field View and Textbook Accounts. *Science & Education*, *12*(4), 387-393. https://doi.org/10.1023/A:1024431115782

Rainson, S., & Viennot, L. (1999) Design and evaluation of a research-based teaching sequence: the superposition of electric field. *International Journal of Science Education*, *21*(1), 1-16. https://doi.org/10.1080/095006999290804

Resnick, R., Halliday, D., & Krane, K. (2006) *Física: volume 3* (5a ed.). Rio de Janeiro, RJ: LTC.

Scaife, T., & Heckler, A. (2010) Student understanding of the direction of the magnetic force on a charged particle*. American Journal of Physics*, *78*(8), 869-876. https://doi.org/10.1119/1.3386587

Scaife, T., & Heckler, A. (2011) Interference between electric and magnetic concepts in introductory physics. *Physics Review Special Topics: Physics Education Research*, *7*(1), 010104. https://doi.org/10.1103/PhysRevSTPER.7.010104

Uhden, O., Karam, R., Pietrocola, M., & Pospiech, G. (2012) Modelling Mathematical Reasoning in Physics Education. *Science Education*, *21*, 485-506. https://doi.org/10.1007/s11191-011-9396-6

Vergnaud, G. (1982) *A classification of cognitive tasks and operations of thought involved in addition and subtraction problems*. In T. Carpenter, J. Moser, & T. Romberg (Eds.) Addition and subtraction. A cognitive perspective. (pp. 39-59). Routledge. http://dx.doi.org/10.4324/9781003046585-4

Vergnaud, G. (1998) A Comprehensive Theory of Representation for Mathematics Education. *Journal of Mathematical Behavior*, *17*(2), 167-181. https://doi.org/10.1016/S0364-0213(99)80057-3

Vergnaud, G. (2009) The Theory of Conceptual Fields. *Human Development*, *52*(2), 83–94. https://psycnet.apa.org/doi/10.1159/000202727

Vergnaud, G. (2012) Forme operatoire et forme predicative de la connaissance. *Investigações em Ensino de Ciências*, *17*(2), 287-304. Recuperado de https://www.if.ufrgs.br/cref/ojs/index.php/ienci/article/view/187

Vergnaud, G. (2013) Pourquoi la théorie des champs conceptuels? *Infancia y Aprendizaje*, *36*(2), 131-161. http://dx.doi.org/10.1174/021037013806196283

Zuza, K., Almudí, J. M., & Guisasola, J. (2012) Revisión de la investigación acerca de las ideas de los estudiantes sobre la interpretación de los fenómenos de inducción electromagnética. *Enseñanza de las Ciencias*, *30*(2), 175-196. Recuperado de https://www.raco.cat/index.php/Ensenanza/article/view/254509/391060

Zuza, K., Van Kampen, P., de Cock, M., Kelly, T., & Guisasola, J. (2018) Introductory university physics students' understanding of some key characteristics of classical theory of the electromagnetic field. *Physical Review Physics Education Research*, *14*(11), 020117. https://doi.org/10.1103/PhysRevPhysEducRes.14.020117

Recebido em: 19.04.2021

Aceito em: 06.12.2021