IMPROVING STUDENTS' MEANINGFUL LEARNING ON THE PREDICTIVE NATURE OF QUANTUM MECHANICS
(Melhorando a aprendizagem significativa dos alunos sobre a natureza preditiva da Mecânica Quântica)

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Abstract

This paper deals with research about teaching quantum mechanics to 3rd year high school students and their meaningful learning of its predictive aspect; it is based on the Master's dissertation of one of the authors (CARVALHO NETO, 2006). While teaching quantum mechanics, we emphasized its predictive and essentially probabilistic nature, based on Niels Bohr's complementarity interpretation (BOHR, 1958). In this context, we have discussed the possibility of predicting measurement results in well-defined experimental contexts, even for individual events. Interviews with students reveal that they have used quantum mechanical ideas, suggesting their meaningful learning of the essentially probabilistic predictions of quantum mechanics.
Keywords: predictive nature of physical theories, probabilistic nature of quantum mechanics, meaningful learning.

Resumo

Este artigo trata de uma investigação acerca do ensino e da aprendizagem do aspecto preditivo da Mecânica Quântica (MQ) realizada com alunos do terceiro ano do Ensino Médio. O ensino da MQ enfatizou o seu aspecto preditivo fundamentalmente probabilístico, tomando por base a interpretação da complementaridade, de Niels Bohr. Nessa perspectiva, foi discutida a possibilidade de prever resultados de uma medida em um contexto experimental bem definido, inclusive, para eventos individuais. As entrevistas realizadas com os estudantes revelam o emprego de idéias concernentes à MQ, apontando para a aprendizagem significativa do aspecto preditivo fundamentalmente probabilístico da MQ.
Palavras-chave: natureza preditiva das teorias físicas, natureza probabilística da mecânica quântica, aprendizagem significativa.

1 – Introduction and research problem

This article describes part of a teaching experiment that inspired the Master's dissertation of one of its authors (Carvalho Neto, 2006). The need to contrast the conceptual bases of quantum mechanics (QM) with those of classical mechanics (CM) motivated us to use a new approach to teaching these theories, seeking to stress the difference between the fundamentally probabilistic prediction of QM, even for individual events, and the statistical predictions of CM. Based on the meaningful learning theory (Ausubel; Novak; Hansen, 1980; Moreira, 1983), we sought from the beginning to anchor the concept of physical prediction in the idea of prediction which already existed in the students' cognitive structure. In this context CM and QM predictions, respectively, were anchored in the prediction subsumer. The purpose of this anchoring was to provide stability and clarity to the idea of prediction underlying physical theories. The idea that physics allows the prediction of results of a measurement in a well-defined experimental context was presented as one of the most inclusive ideas of physics (Osnaghi, 2005). The focus adopted, according to the classification proposed by Greca (2000), Ostermann & Moreira, (2000), was to present QM without linking it analogically to CM. In line with this proposal, we chose for discussion a groundbreaking experiment (Tonomura et al, 1989), involving interference with a single "particle'', seeking to provide the students with an intuition that would enable them to adequately perceive actual quantum phenomenology (Greca e Herscovitz, 2001).

Our teaching experience in both grade school and college for over a decade led us to observe the students' trouble in grasping the predictive aspect of Physics (Freire et al, 1995). The fundamentally probabilistic predictive aspect of QM, for instance, has mistakenly been attributed by many high school students to people's ignorance regarding quantum systems. Before studying QM, many have no idea of the differences between quantum systems and deterministic dynamic systems with poor predictability.

A more systematic observation of prior QM knowledge subject to conceptual evolution took place in 2004 in research done on third-year high school students at the Colégio Antônio Vieira (CAV) in Salvador, Bahia, Brazil.[1] Initially the students did not recognize the existence of a fundamental limitation in the use of the concept of trajectory in the atomic and subatomic domain. Additionally, it was interesting to note that the students showed a lack of awareness about the notion of classical determinism. Based on their knowledge as shown in our research, we decided to design a teaching proposal to present the predictive aspect of CM and of QM. The new approach to CM is not the focus of this article, having been presented in a different article. (Carvalho Neto, Freire Jr., Silva, J.L. 2008).

The research problem which presented itself, then, was: What teaching strategy could enable students in the last year of High School to grasp QM's predictive aspect, even for individual events?

This study's general and most widely encompassing goal was to give explicitness and emphasis, at the high school level, to the predictive aspect of quantum mechanics. The specific goals proposed were: a) describe the cognitive structure of high school students regarding the meaning of QM prediction and the meaning of probability in the context of this scientific theory; b) design, implement and assess a teaching strategy for approaching QM's predictive aspect, even for individual events.

We did not seek to make a simple change in the pupils' cognitive structure. Moreira (apud Peduzzi, 1999), drawing from the theories of Ausubel, Novak and Gowin, had warned of the difficulty of effecting conceptual change because alternative concepts are also the fruit of meaningful learning and therefore resistant to change. In line with this thinking what we sought was strategies for the learning of shared meanings in a scientific context and not the "abandonment of alternative meanings, but the acquisition of the awareness that such meanings are erroneous in the context of science."

[1] In Brazilian schools, students must study physics during high school, since this subject is an obligation for entering the university. The exam, called Vestibular, is a set of tests, in which there are questions relative to Portuguese, mathematics, chemistry, biology, history, social studies, English or Spanish, literature and grammar topics. The 3rd (third) year of study, which is the last year of high school, represents a process of a high level of tension and emotional stress. The entrance process, Vestibular, is almost the only way to achieve the "status" of being in college.

2. Study design, material and methods

The whole situation cited above created the conditions for a pedagogical challenge to be taken on and for it to still be a research project. It was a qualitative study based on participative observation, collection, analysis and interpretation of information (Bogdan; Biklen, 1994). Since it was a descriptive-analytical study, it aimed to utilize knowledge produced for the purpose of intervening in situations perceived as subject to conceptual evolution, where the researchers and their subjects voluntarily take a reactive position. This kind of study is also called research-action-intervention. On the other hand, the interpretative aspect of the study needs to be stressed because the interviews were analyzed and interpreted. For each teaching experiment, there were pre-tests, prior interviews, a mini-course and exit interviews.

The sample consisted of ten third-year high school students at a private school in Salvador, Bahia, Brazil, five during the exploratory phase of the study in 2004, and the other five during the new approach to QM teaching in 2005. This choice was due to the ease of access to the school and to the third-year class by one of us (Carvalho Neto), who acted as researcher-investigator; and also due to the support given by the school to this experiment in QM teaching. The students accepted the invitation by the teacher-researcher, they were informed of the research goals, and they signed informed consent forms giving authorization to publish their statements.

Taking into consideration that the qualitative research process presupposes different possibilities for programming its execution, we planned two moments of study which were distinct, yet internally correlated. The first moment, carried out in 2004 as a 20-hour mini-course, can be defined as exploratory research because its purpose was to identify preexisting ideas in the students' cognitive structure regarding the predictive aspect of QM. This inventory – a synthesis of the students' prior knowledge about the subject was fundamental for defining the strategies chosen for the second moment, which was in 2005, keeping in mind the search for the answer to the problem formulated in the study. The pre-course interviews, teacher-subject dialogs, were numbered from 1 to 5, and the oral discourses by the students were arbitrarily codified as A1, A2 and so on, followed by 2004 or 2005, the years that the linguistic material was collected. The code A1-2004-a, for instance, refers to Student No. 1 of the 2004 school year before the instruction began and the code A1-2004-p refers to the same student after the instruction.

3. Description of the mini-course

The 20-hour QM course was given in December 2004 when the third-year high school program was completed. The great source of tension we had to face in 2004 was the contradiction existing between the content of the quantum physics program for the *vestibular*, university entrance exam, (centered on Bohr's atom and the photoelectric effect) and the fundamentally probabilistic predictive QM content, which includes the Principle of Uncertainty. We had started attempting to overcome this problem in 2004 by including the aspects of the new quantum mechanics through a historical approach. However, it wasn't until 2005 that we planned a mini-course with the goal of responding to the above mentioned research problem. Since we couldn't stop covering the topics of the old quantum mechanics in 2004, we adopted the strategy of presenting it as a subject that lacked a more general predictive aspect centered on semi-classical ideas in contrast with actual quantum phenomenology, described only by the new QM, produced between 1925 and 1927. The focus was on the difference between QM ideas and classical physics ideas, instead of on analogies. A summary of the topics covered in the 2004 course is shown below.

3.1 The photoelectric effect

We started by revisiting the concept of electromagnetic radiation from the perspective of the classical concept, reminding the students that diffraction and interference of light are typical phenomena of waves. Next we discussed the interaction of electromagnetic radiation with matter, stressing the inadequacy of the wave model in the description of emissions phenomena. For this purpose we discussed both qualitatively and quantitatively the results involving the interaction of ultraviolet light with a metal photoelectric cell in a vacuum. We analyzed graphs of the intensity of the current collected by the circuit due to the potential difference between these plates, pointing out that the cut-off voltage does not vary with the intensity of the monochromatic light used in the experiment (something unexplainable from the electromagnetic theory point of view). We also analyzed graphs of current intensity as a function of cut-off voltage considering lights of different frequencies – yellow, violet, and ultraviolet – stressing the fact that cut-off voltages vary with their frequency, also unexplainable from the perspective of electromagnetism, which by the way also failed to predict an almost instantaneous emission of photoelectrons. This was the common thread that gave the theoretical support to point out the three problems that remained unsolved until the beginning of the 20th Century. As a source for planning how to teach the photoelectric effect, we used Moysés Nussenzveig's textbook (Nussenzveig, 2002, pp. 249-254).

3.2 Bohr's atom

The idea that accelerated electrical charges radiate was the starting point for showing the inconsistencies between Rutherford's atomic model, properly supported by experimental results, and classical electromagnetism. It was in this context of the failure of classical physics to comprehend atomic stability that we introduced Bohr's postulates and their consequences. We also showed that that quantization of the angular momentum, applied to a hydrogen atom, considering the Coulomb force and the law of the conservation of energy, led to quantized energies as well, expressed as $E = -13.6 \text{ eV} / n^2$.

The idea that electrons could change "orbits", emitting or absorbing discrete quantities of energy, was also introduced. The Compton Effect, on the other hand, was not discussed in quantitative terms. We merely commented that the American scientist proposed to attribute a linear moment to the quantum of radiation, in addition to $E = hf$ energy, and that he thus established the idea that radiation could in some experiments manifest properties of corpuscles, for instance what happens when x-rays are scattered through graphite powder.

3.3 Wave-particle duality

The idea that matter can exhibit wave properties, as proposed by De Broglie, extended the wave-particle duality of electromagnetic radiation to matter. This brought the realization, suggested later by Niels Bohr through the concept of complementarity, that the description of phenomena, involving both those of radiation and those of matter, will only be complete through the complementary and mutually exclusive use of wave and particle models.

The textbook initially used for preparing the presentation of wave-particle duality was Eisberg's (1994, pp. 94 and 95). Some problems were solved in order to show that the very small value of Planck's constant prevented the perception of the wave aspects of macroscopic matter, keeping in mind that wavelength λ associated with matter has a value of h/p, where ''h'' is the Planck constant and ''p'' is the linear moment. The application of this conceptual innovation in building the electronic microscope was discussed in class, comparing the wavelength associated with visible light to the wavelength associated to an electron, relating wavelength magnitude to the power of resolution.

3.4 QM formulated from 1925 to 1927

In 2004, to complete the 20 hours of the course, three hour-long class sessions were reserved for the discussion of the conceptual innovations that established quantum mechanics between 1925 and 1927, using a historical approach.

For theoretical support we used the whole chapter entitled "Enfim a Física Quântica" from the book *O Universo dos Quanta* (Freire Jr. and Carvalho Neto, 1997). Our aim was to teach that there was a fundamental limitation in the simultaneous use of the concepts of position and velocity in the atomic and subatomic domains and that this limitation was expressed by Heisenberg's Principle. It was within this context, with a historical approach, that we presented the break between classical determinism and the probabilistic feature of the theory concluded in 1927.

The course ended with the statement that we are "the contemporaries of an unconcluded controversy'' (Freire Jr., 1999).

4. Discussion of exploratory phase results

In the process of analyzing and interpreting the data, we sought to understand the students' conceptions about the use of probability in QM. In the preliminary interviews, none of the five students made the distinction between the meaning of probability in the macroscopic domain and probability in the microscopic domain.

Student A2-2004-a, for instance, stated the belief that the use of probabilities occurred in both the macroscopic and microscopic domains, but did not analyze the epistemological differences. He argued when discussing the behavior of a ball and an electron that "a ball thrown from the top of a building can describe different trajectories depending on how it is thrown, on initial velocity and on its obstacles. In the 'micro', an electron can also receive influences from its medium (electrical or electromagnetic forces) and fall into the world of uncertainty. Therefore, probabilities can be applied in various microscopic and macroscopic contexts." One can see that this student makes no distinction at all between a "classical uncertainty'' and a "quantum uncertainty."

In a similar vein, Student A1-2004-a states that: "probability is used in classical physics for every phenomenon which is not described in an ideal way. I think it is used more often in quantum mechanics.'' It can be observed, however, that the student does not make explicit an essential difference between the use of probability in the macroscopic and microscopic domains.

Student A3-2004-a, in a similar argument, asserts that "there are physical systems which do not provide conditions which are ideal for fitting into formulas and theories. Therefore some approximations are made and probabilities are used."

Student A4-2004-a states, in turn, that we use probabilities "when a body is subject to several forces which are not constant, obliging the scientist to calculate probability." This student mistakenly associates the use of probability with the absence of the predictive characteristic of physical theories. He reasons that "probabilities in physics are only applied when it is not possible to determine in advance through calculations if a physical phenomenon can occur."

Student A5-2004-a states that we use probabilities "when we cannot obtain proof through the experiment or when we are unsure of the result," reinforcing again the idea, though posited implicitly, that probability is the expression of a lack of exact knowledge.

The analysis of the pretest also provided important information about the predictive aspect of Physics. Although the students have in their cognitive structure a stable clear organized idea of prediction in the ordinary sense, for instance that it is possible to predict if it will rain tomorrow, they were unaware that prediction is a characteristic of physical theories.

4.1 Analysis of the Interviews

The process of analyzing and interpreting the meanings constructed by a universe of five students, who were interviewed in the period after the mini-course in 2004, resulted in the grouping of these meanings by category, depending on their similarities and differences, keeping in mind that each category was defined by considering the meanings that were similar to each other. This categorization came from reflecting on, analyzing and interpreting the meaning constructed on the fundamentally probabilistic predictive aspect of QM.

The process for grouping the meanings by category was developed while observing the following criteria:

a) the description, analysis and interpretation of single interview, giving recognition to each relevant section of what was said by the student, as it appears in the transcribed dialog between the interviewer and Student A5-2004-p;
b) the analysis and interpretation of the statements by the other interviewees, recording the similarities and differences between shared meanings, seeking to define categories that would include the variety of meanings constructed by the students;
c) the synthesis of the meanings identified in the interviews, by grouping them in broader categories.

The goal of the analyses presented here was to seek solutions to the research problem, as one can observe below.

Interview A5-2004-p began with the interviewer's request that the student describe in a critical way the main conceptual innovations coming out of QM. He answered with assurance that: "QM rejects the concept of trajectory and uses probability as an obligatory factor." He refers to the inadequacy of the simultaneous use of the concepts of position and velocity in the atomic and subatomic domains, arguing that Heisenberg "says, in his principle, that either you know the velocity of the electron or you can determine its position. If you know one of these two elements you will be unsure of the other. You would not be able to know the electron's velocity and position simultaneously."

The student also acknowledges that there is no analogy between the deterministic macroscopic dynamic systems and quantum systems, since the concept of trajectory is valid only for movements on a macroscopic scale. As evidence of this notion, let's look at the following excerpt.

> **T***: You mentioned Tsunami… if you take a little ball and place it in a whirlpool… a very turbulent one… do you think it's possible to treat this little ball deterministic manner?
> **S**:** Yes, it's possible.

T: Do you think there's a similarity or a difference between the little ball in a turbulent fluid... if you compare it with an electron travelling from one layer to another? Do you see more similarity in these phenomena… or do you see more differences?

S: differences.

T: What is, in essence... the difference?

S: because an electron doesn't follow a trajectory… and the little ball does.

T*= Teacher

S**= Student

The dialog above signals an advance in the comprehension of the break from classical determinism in the atomic and subatomic domain. However, this same pupil is unable to perceive the predictive aspect of QM. The pupil incorrectly concludes that quantum uncertainty implies the absence of this theory's predictive aspect, as the answer proves.

T: Is quantum mechanics a predictive scientific theory?

S: quantum mechanics? I don't know. Is it?

T: Come on, let's reason together. I'll try to help you, not by answering the question… but by helping you to do your own reasoning.

S: Can I guess?

T: Yes, you can. Tell me.

S: I think that quantum mechanics is not predictive… starting from the moment that it is working with probabilities…it is trying to give a probability…not a certainty… so it cannot predict.

This pupil's mistaken idea that QM is not predictive is associated to the idea that QM is not efficient. The pupil says that it (QM) "is working with probability. It won't be able to predict efficiently." In this pupil's opinion, QM has no practical sense.

To conclude the analysis of the interview with Student A5-2004-p, it should be pointed out that a pragmatic tendency was perceived, as one can observe in the description and analysis in the following excerpt. He says that the "comprehension" of this theory is only possible through pragmatic acceptance. He makes use, as an analogy, of the idea implicit in the Theory of Special Relativity, which refers to light's constant velocity, as a non-intuitive idea, and says that in order to use the quantum theory, in turn, one must "accept" the theory, stating that otherwise "he wouldn't be able to understand", referring to the limitations of uncertainty implicit in Heisenberg's Principle. Let us see:

S: I believe that they established that as a principle.

T: And how do you regard this? Do you…do you think this is something…uh…that this bothers you somehow?

S: In a way... uh... I try not to let it bother me... I try to accept this fact.

T: You try to accept it. Why?

S: For instance…uh…relativity…states that the velocity of light is always a constant…if you go one way or you go the opposite way…it is still a constant… and this is sort of hard to accept…so what do you do…you simply have to accept it and see what you can do to take advantage of it…like Einstein…so I try to do the same thing…I need to get used to it and follow other people…of course…so, I try to accept quantum physics without questioning it too much because otherwise I wouldn't understand at all.

What stands out in an interpretative reading of this interview is that the student, although he understands the limitation of the concept of trajectory in the atomic and subatomic domain, is unable to properly grasp QM's predictive aspect.

4.1.1 Interviews of A1-2004-p to A4-2004-p

In this section the meanings constructed by the students in interviews A1-2004-p, A2-2004-p, A3-2004-p and A4-2004-p will be analyzed and interpreted. The goal of the analysis, pondering QM's predictive aspect, the same theme approached in Interview A5-2004-p, is to broaden the comprehension of the perceptions and meanings revealed in the analysis presented in the previous topic, making possible broader interpretations.

The conflicting idea that QM is not a predictive theory appears in an explicitly mistaken way in the statement by Student A2-2004-p as well:

S: a conflict?...my doubt… my…this doubt…this conflict comes from a mind loaded with Newtonian Mechanics…because if you reject trajectory…what are you going to predict? If you don't predict velocity… and space at the same time? What are you going to predict? What good is this theory? Uh... I don't...I don't know what to say.

Thus one can observe a reactive and conflictive position, signaling his doubt about quantum theory prediction and revealing that QM prediction is not integrated into his cognitive structure.

Another important aspect to stress is that this student, although he has not properly grasped QM's predictive aspect, showed that he knew the difference between the intrinsic use of probability in QM and the eliminable use of this concept in CM. In order to illustrate this perception, part of the dialog is shown below.

> **S**: I spoke of… about classical physics…Now I'm going to talk about quantum physics…well, probability… I think…right…I…I…I think like this, that probability in… in the field of quantum physics…it is…it…it is inherent to…to…to…to its study…because when you use probability to…to…to say, for example, the position of an electron…you…it's not that you lack information…it's that…uh…it has a…its position is…then…is then…is really…not well defined…you understand? So…it's not because of a lack of information…it's already something that…is…already…already inherent…that's why I used this word…it's something inherent…to…to…
> **T**: in the macroscopic world…right? …the probability to come up heads is 50%... don't you use that, too?
> **S**: Yes…but…that probability there you use because you don't know about…about what might interact with…with that coin there…but if you have everything…have everything that could interfere with the coin's trajectory, you don't need to use probability…understand?
> **T**: What do you mean, you don't need to? Why is it a matter of ignorance? Or do you mean you don't need to because of a problem of uncertainty?
> **S**: Because of the problem of uncertainty?
> **T**: Who expresses this uncertainty?
> **S**: Heisenberg's principle.

Student A4-2004-p, in contrast with the other students we studied, showed that he had constructed an incorrect meaning about the role of probability, since he attributes the probabilistic aspect of QM as something lacking in the perfection of the theory.

> **S**: it was…you know? It might be a weakness in the theory…but I believe it's because of technological and scientific inefficiency…because I think it may be that 'x' years from now…I don't know how many years, a trillion years…it'll be…it'll be that way…I think it'll be like that.
> **T**: OK, now I understand your…your point of view…but…do you think that the quantum theory…that it is a complete or an incomplete theory?
> **S**: incomplete…uh…developing…it can be perfected.

It is evident that in the student's view the lack of information is a characteristic trait of QM. Student A3-2004-p, like Student A5-2004-p, provided a satisfactory explanation of the limitation of determinism on the atomic and subatomic scale, adding that it no longer makes sense to speak of initial conditions in a QM context. He clearly explains the incompatibility between the concept of trajectory and Heisenberg's Principle. In reference to the descriptions on the atomic and subatomic scale, he states that: "it isn't even coherent for you to speak of initial conditions because the initial conditions are initial velocity and initial space. Not unless you have, in the case of the electron, knowledge of the initial space will you have an infinite uncertainty of the velocity, at that moment, and vice-versa. So that's why the concept of trajectory can be rejected."

In an attempt to more clearly understand the meanings in his statement, he was asked:

> **T**: Just to make this clear…uh…are you saying that the initial conditions are defined but that we don't know them?
> **S**: no…
> **T**: or that they are undefined?
> **S**: they're undefined.

It can be seen in the statements that this student also realizes that the use of probability in QM does not reflect human ignorance, as illustrated in the following excerpt.

> **T**: Right...Now...this statement you're making…is it a reflection of our ignorance? Of not knowing these magnitudes…in this case…that you cited…space and velocity? Is it a problem of ignorance? Is it a technological problem…this impossibility of measuring?
> **S**: no…it's because it just does not appear

One can observe in the interviews analyzed that the five students showed an understanding of the incompatibility between the concept of trajectory and the principle of uncertainty. They all acknowledged that QM does not prohibit the measurement of the position or quantity of motion, in isolation, but the simultaneous measurement of both. There seems to have been meaningful learning about the principle of uncertainty since the ideas of position and momentum, which pre-existed in these students' cognitive structure, interacted in a substantive and non-arbitrary manner with the new information, that it is not possible to measure these two magnitudes simultaneously.

4.2 Conclusions from the analysis of the exploratory phase results.

The results led to the following conclusions:

1. Most of the students understood that the use of probability in QM is not the reflection of human ignorance the way it generally is, on the macroscopic scale.
2. Some students see in the non-eliminable use of probabilities the mistaken idea that QM is not a predictive theory. One consequence of such results is that the students, while understanding the limitations of classical mechanics in the atomic domain, cannot grasp a quantum perception.

The critical understanding of this conceptual limitation, which we observed in the exploratory phase, was utilized as a guideline for definition and strategies adopted in 2005.

5. The new approach to QM

The quantum mechanics mini-course in 2005 was planned in order to permit a response a solution to the problem of this study. We revisited the idea of the physicist Richard Feynman (1969), that the "mystery" of QM can be understood through the discussion of the double slit experiment. The construction of meanings about QM's essentially probabilistic predictive nature had the analysis of the double slit experiment with electrons as its common thread. We attempted to teach complementarity by using the concept of phenomenon created by Bohr. We stressed that a comprehensive understanding of both electromagnetic radiation and of matter (such as the electrons in these experiments) was only made possible by resorting to both concepts in a complementary and mutually exclusive way. As an integral part of the course we showed and analyzed a film, in real time, about an experiment done in Japan (Tonomura, 1989) in which there is a pattern of interference, even with the individual arrival of electrons. What particularly stood out was that it was a wave phenomenon that was being studied, even for individual events. Additionally, it was argued that other systems subject to quantum treatment, such as light, which in experiments with the photoelectric effect exhibit corpuscular behavior, would also exhibit a similar pattern, under experimental conditions analogous to those presented in the film with electrons, even when launched individually (when it is not known through which slit it passed). Next, this phenomenon was treated in quantitative terms by drawing a probability curve (p) as a function of position (x), considering this experimental condition. Right after that an analysis was made of the appearance of the corpuscular phenomenon when it was detected which slit the electron had passed through, where the interference pattern disappeared. Thus we approached the wave-particle duality and Heisenberg's Principle, not as isolated topics but as expressions of complementarity. The essence of this approach was centered on the idea that the principle of uncertainty was an expression of QM's correct and complete probabilistic prediction, according to complementarity.

5.1 Analysis, discussion and interpretation of data: results obtained and conclusion after the new approach to QM

The new approach was used with all the third-year students in 2005. However, a "universe'' of only five students was studied. A code similar to that of 2004 was used to identify each student interviewed. We stayed with the idea of presenting excerpts of an interview and then grouping the similarities to and differences from the meanings constructed by the other four students.

The first result that stands out (from the interview with Student A1-2005-p) is the accurate perception of the meaning of the quantum phenomenon. He acknowledges that the observation of the electrons plus the experimental conditions established by the observer will define the phenomenon as either wave or corpuscular. He makes a correct reference to the electron interference observed by Tonomura by saying that:

S: Yeah! The experiment consists of emitting the electrons one by one and then there starts…starts to appear…a pattern of interference in the screen, directly behind the two slits. And this experiment serves…in the…according to the interpretation of complementarity to precisely show the wave-particle duality. Because the electron…that…in that interpretation of waves, it doesn't matter if there is one electron or several electrons. In other words, it's impossible to know which slit the electron went through because if it had gone through just one slit, it's impossible for it to interfere.

The interviewer presses him on the question about the nature of the phenomenon, stressing the experimental condition of the electrons arriving individually. Let's take a look.

T: ...if I do this experiment (with slits), and you have the screen detecting the arrival of three electrons. One comes, another comes, the third one comes. This phenomenon, is it wave? Is it corpuscular? Or is it both? According to the orthodox interpretation by...by Bohr.

S: Ah! Yeah! The two of them...impossible, according to this interpretation. Because the two events are mutually exclusive…By the way, that was a doubt I cleared up when I watched the film. Because when you emitted (just one electron, there's no way for you to observe a pattern for just one electron. But conceptually, like that, I…I think that, conceptually, it's a wave phenomenon…

The assessment of meaningful learning also took into account the dual aspect of electromagnetic radiation. It was considered essential to raise doubts, over the course of the interview, about the idea that light can also exhibit corpuscular or wave properties, depending on the experimental conditions imposed by an observer who prepared a given system. For this purpose we proposed an experiment of imagination (thought experiment) with individual events, involving photons. In order to illustrate once again meaningful learning about the concept of phenomenon, we will reproduce another excerpt from the dialog which refers to an experiment involving very weak light, consisting of photons arriving one by one. The dialog is in progress:

T: Let's do this; let's repeat this experiment that we're discussing here, except with a very weak light, down to the point that I can shoot it photon by photon…this phenomenon, would it be…would it be wave or corpuscular?

The student's answer was:

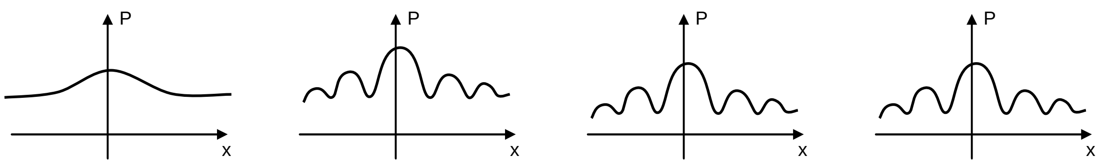
S: "...according to that argument by De Broglie, that to all bodies there can be…Yeah! Associate a wavelength, the phenomenon would be wave as well. I think that one there would be analogous to the phenomenon of the electrons."

The wave-particle duality discussion paved the way to a solution to the research problem. The idea that there is a fundamental difference between basically probabilistic quantum predictions and classical predictions (the former being incompatible with the concept of trajectory) explicitly appears in the same student's statement. Right from the beginning, he acknowledges that physics is a predictive science and notes that "physical prediction is the anticipation of phenomena based on the corresponding theory." Regarding the differences between CM and QM, he argues that "the exact break with CM was discovering that subatomic particles don't obey certain laws, don't obey the concept of trajectory and including probabilities as a…a characteristic inherent to the…to the theory…an indispensable characteristic of the theory,…”. Next he refers to Tonomura’s experiment and says that:

> **S:** “It’s because when you learn that the trajectory...that there’s no trajectory for the electron, you…all your concepts collapse…But I began to, let’s say it this way, to believe in…the probabilistic interpretation with…with that experiment, with the film about that experiment about the…about the two slits that were launched/…that several electrons were launched and then there appeared, really, a pattern of interference for the electrons…Before…before the course, before the… taking the classes, I thought the quantum theory, specifically, was not a predictive theory because it didn’t say exactly where…what position and what velocity. I thought that probability would be something opposite from…from a predictive theory. But as I already…already answered before that one of the characteristics inherent to all physics theories is the predictive characteristic, I think that the…the quantum theory is also predictive because it foresees the probabilities of the…of the phenomenon. For instance, in the pattern of interference, the probabilities of the…of whether…Yeah! ...the probabilities of your finding the electron in a certain position.

It can be seen that the idea that the use of probability is intrinsic to quantum theory is linked to this student’s perception that QM is a predictive theory. Thus there was a subsumption of quantum prediction into his cognitive structure. Meaningful learning was possible only possible because in the student’s cognitive structure there was the idea that physical theories are predictive, which in turn was only possible because of the previous anchoring of the concept of physical prediction in the subsumer prediction. We believe that the development of the teaching material containing the idea about common-sense prediction such as weather forecasts, which worked as an advance organizer, has helped to give, prior to the intervention, more stability to the subsumer prediction, an idea in the student’s cognitive structure which was already present and available.

An additional result was perceived. The student showed that he knew how to draw graphs of probability distribution for gun bullets, classical waves, electrons and photons, launched one by one, with both slits open. The graphs below, which the student commented on in the interview, were drawn during the written assessment the day before the interview.



When asked to comment on the correctly drawn graphs, he explains that: “…The curve of the…of probability in relation to…to the X (X, the position), in the…with…with electrons is very similar, I mean, it’s the same, I’d say it’s the same curve that…when…as if the phenomenon were wave…That just reinforces the idea that the phenomenon is wave.”

In an attempt to perceive if the student realized that a quantum corpuscular phenomenon does not refer to a classical particle, the researcher inquires:

> **T:** Why is it that you cannot call the electron a classical particle?

The same student, then, answers:

> S: Because of… of the uncertainty of your knowing the position and velocity at a given instant.

Another important result was the student's perception that the classical concept of state, which is preserved in Bohr's Atom, proposed in 1913, is conceptually different from the QM state of wave function. This evidence was qualitatively observed in the following excerpt:

> **T:** Based on the concept of classical state and quantum state, evaluate how far the atomic model proposed by Bohr in 1913 moved away from the quantum theory formulated from 1925 to 1927.
>
> **S:** Well, that was exactly...was exactly some doubts we had about the…Bohr's model, but he came before quantum physics. That's why it has some characteristics that move away. For instance, when you start to calculate, you use the resulting force as electrical force, that is, you use Newton's Second Law to describe the…the trajectory as circular, then you associate to it a pair of velocity and… and position which are well-defined.

One should note that another result is that this student, although he has understood the probabilistic trait of QM, he shows a strong tendency to consider this positioning as temporary. This supported the investigator's presupposition that presenting the interpretation of complementarity, far from stimulating random personal preferences, helps the student sharpen his sense of critical thinking. Let's see an excerpt that gives evidence of the student's viewpoint:

> **S: "**And, about a doubt I still have. It's not exactly a doubt, but it's a discomfort I still feel about probabilities being intrinsic to quantum mechanics. I still…Yeah! ...I'm still…tending to analyze the…the…or…other interpretations with…let us say, with both eyes. Because it's…it's a support you have, when you've always learned deterministic concepts, for you to suddenly see a theory that's totally probabilistic it's…it's a good support for you to base things on, like, on the…on Einstein or on some other existing theories that want to refute this concept of intrinsic probability.

Next we will analyze and interpret the meanings constructed by Students A2-2005-p, A3-2005-p, A4-2005-p and A5-2005-p. The analysis, pondering QM's predictive aspect, the central theme of the predictive aspect approach used in Interview A5-2004-p, has the goal of broadening the understanding of the perceptions and meanings revealed in the analysis presented in the previous section, making possible broader interpretations.

These students, in general, like Student A5-2005-p, constructed correct meanings about the essentially probabilistic predictive aspect of QM. The main difference, though, is that one of the students, (A3-2005-p), refused to understand Physics without images. (CARVALHO NETO, 2006)

As evidence of meaningful learning about the correctness and completeness of QM, according to the interpretation of complementarity, we must include an excerpt of Student A4-2005-p's statement. This student, referring to Heisenberg's principle as the complete expression of a quantum physical system in the atomic and subatomic domain, states that it is possible to "predict the states of a particle…, let us say, predict the probabilities. I think that the relationships of uncertainty help make the quantum theory complete."

Reinforcing the idea of meaningful learning about quantum mechanics' predictive aspect, the student relates probabilities to the principle of uncertainty, applied to individual events. Note that the student has a clear idea that probabilities in the quantum world do not reflect our ignorance.

> **S:** No. I mean, yeah, it's…no…if…Yeah!...intrinsically, it doesn't… this pair doesn't exist, you'll always have an uncertainty about…between position…the position and the velocity. And, regarding the...the probabilities that we use in CM, it's precisely out of ignorance, out of not knowing, or being very difficult for you to say, exactly, where…what this is…what these characteristics are of the…of the body.

Student A3-2005-p, in turn, referring appropriately to the differences between CM and QM states that "in quantum mechanics, you don't have the same…you can't talk about trajectory like you do in the classical mechanics of Newton. There's nothing similar to…a person may not have learned anything about CM, but if…if someone comes up to this person and teaches QM and says this is how it is, no…it doesn't make much difference."

When asked to explain more about the conceptual differences, the student states that:

> **S:** ...the electron, like the photon, like any other subatomic particle, does not at any instant have a well-defined position or velocity. That's how I understand it. And this incapacity is not a defect; it's a characteristic of quantum mechanics. Those velocities and those positions are not well defined.

The student further stresses that probabilistic prediction holds true even for individual events, saying that:

> **S**: The difference is that quantum mechanics has to use probability in its calculations. And the Newton type gives sort of a one result only. If you know the initial condition, then you make the calculations, and there you have it, no matter how hard it is. And not quantum mechanics; you have to use probability to know what it is that will happen. (A4-2005-P)

> **S**: It would be probabilistic...It doesn't matter if you throw a thousand or one, it will be the same thing… Won't it?!...

The conversation goes on, now in a more specific vein, about the research problem. Making reference to Young's experiment, the interviewer asks the student to speak freely about what he learned.

> **S:** Good thing... Right?!...Yeah!...first this wave-particle duality, it was an idea that arose from…after quantum mechanics. Before that it was thought that it was either a wave or a particle. That's very clear. After…the fact of setting up the experiment determining a type of phenomenon is also something that quantum mechanics brought,

The interview with Student A3-2005-p, from the beginning, is centered on a radical refusal to think about physics without images, an idea which has supporters and non-supporters in the science area. The student's inflexible position was extremely important for bringing up a question that has been around since 1927, when the physicist Lorentz stated that he could not imagine the idea of physics without images, saying, "We have always wanted, up to now, to form images from ordinary notions of space and time. These notions may be innate."

This student, in contrast with the other four, refuses to think about physics without images. Referring to the matter of the above mentioned test, his statement differs from that of the rest of the students. He says that, "I can even think, I don't know, kind of as if it were a classical wave coming by, when it struck it would each…each point would be a new wave front and it would be… that's exactly where the problem is. It (the electron) is not a…not a classical wave which when it arrives there, it strikes and becomes a little ball. I…can't understand. It's something…it's as if they were saying two things that would be contradictory. I…can't understand. No…I can't…it doesn't work for me.

Seeking more information, the interviewer proceeds: "Listen, and the fact that the results of the experiment agree with the theory, doesn't that…for you isn't…isn't that enough?" The student replies: "No. Because, it's like I say, I need to understand that." It should also be noted that when referring to probability curves like the ones in *Lectures*, by Feynman, the student reveals that the results of the experiments conflict with his intuition. He says that, "the graph of probability for the…for the…with the gun bullet, I can understand. Fine. Now the graph with electrons? I know that that happens. I know that they did an experiment that ended up working out, but if...if somebody..."

The analysis of the interview with Student A4-2005-p is that, like most of the students observed, he understands in a meaningful way the predictive aspect of the theories of physics, and in a special manner, the essentially probabilistic predictions of quantum mechanics. When referring to the situation in which light is used to detect which slit the electron has gone through, this student argues that, when establishing the experimental context, it's possible to predict whether there will or won't be interference with the electron in individual events.

> **S**: As far as I know, when the electron goes through the slit with the light turned off, it shows an interference fringe. And I know that they did that before the experiment. That's because the experiment didn't take place until 20 or 30 years later.

6. Conclusion

The solution to this research problem, collected from interviews, generally points to the occurrence of meaningful learning about the essentially probabilistic predictions of quantum mechanics. Evidence of this learning was the acknowledgement by the students we observed of the possibility of predicting measurement results in a well-defined experimental context in the atomic and subatomic domain. These students acknowledged, for instance, that the act of preparing and observing a given quantum physical system, in a given circumstance, defines the phenomenon as wave or corpuscular, from the viewpoint of the interpretation of complementarity by Niels Bohr. In particular, after watching the film about the double-slit experiment done in Japan (Tonomura, 1989), they perceived that the lack of definition about which of the slits the electron had gone through is compatible with the idea of an interference, even for individual events, and that the definition of which slit the electron went through is compatible with the idea of a corpuscular phenomenon. There is also, in line with these results, the perception that the electron is neither a classical particle nor a classical wave. When they drew the curve of probability as a function of position, most interviewees acknowledged that these predictions did not represent classical statistical predictions, but rather irreducible and uneliminable probabilities which, by principle, expressed a reality described by the quantum theory, according to Bohr (1935). It is within this context that the students qualitatively perceived the existence of a difference between the quantum physical state described by the psi function and the classical state defined by position and velocity, which was facilitated in our view by the effort to stress the differences between the two theories instead of pointing out misleading analogies between them. It may be because of all of this that the new approach to QM helped bring the students nearer to understanding actual quantum physical phenomenology. The acquisition of this phenomenology occurred in one student together with a refusal to study physics without images. This student stated that he could not comprehend physics that does not deal with images. This result, which was not researched in this study, warrants later investigation. More precisely, it needs to be investigated how far the learning of strictly quantum concepts can or cannot get around the fact that in the interpretation of complementarity, QM by definition does not supply such images.

Meaningful learning about the predictive aspect of the theories of physics was only possible when there was a subsumer, that is a previously established stable idea about physical prediction anchored in a pre-existing idea in the student's cognitive structure, that of "common sense" prediction. Lastly it should be noted that in at least one concrete case, one student (A1-225-p), in spite of having expressed good evidence of meaningful learning about the concepts presented, particularly probabilistic prediction, also expressed a clear tendency to consider probabilistic prediction a temporary positioning that may be refuted in future scientific development. Now it is true that this student was questioning the completeness of the QM proclaimed by complementarity. From that perspective, it raises the issue of knowing how much the learning of QM concepts can be dissociated from a reflection about the controversy related to the interpretation of this scientific theory. This issue was not explored in the present investigation, but it warrants further study.

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